

Feeling Textures through a Probe: Implications of Data Normalization Procedures for Preservation of Quadratic Parameters in underlying Magnitude Perception Process

Addendum to Klatzky, R. L., Lederman, S. J., Hamilton, C., Grindley, M., & Swendsen, R. H. (2003). Feeling textures through a probe: Effects of probe and surface geometry and exploratory factors. Perception & Psychophysics, 65, 613-631.

This addendum offers a proof that the treatment of the data in this paper will preserve critical parameters of a quadratic function that is assumed to underlie the subjects' responses. The proof adopts a model stipulating that a common quadratic applies to all subjects, within a multiplicative constant. That is, subjects differ from one another only in terms of the value of a multiplier that is applied to the output of the underlying sensory function. Across different conditions, the function representing the sensory response changes, but subjects maintain the same multipliers. For example, if Subject 1's response is twice Subject 2's response in condition a, then the 2:1 ratio is maintained as well in conditions b, c, etc. To summarize the implications of the proof, under these assumptions, the quadratic and linear term of the underlying function are preserved absolutely, and the constant is preserved relatively across conditions. As a result, the data analysis preserves absolute curvature and the location of the peak of the quadratic, as well as the relative height (difference in height between pairs of conditions within a single experiment).

If, instead, the proportionality relation among subjects varies with conditions, the quadratic and linear terms -- hence curvature and peak location -- can be preserved, but not the height of the function, even in relative terms. Below, we show that alternative treatments of the data will similarly preserve curvature and peak location, but not relative height.

Model

In a given condition (m), for a stimulus plate with a given interelement spacing value (s), a quadratic function, $q(s, m)$, generates an internal sensory response representing the plate's roughness. The quadratic function operates on the log of s , and the subject's response (X) is the exponential of its output, multiplied by a proportionality constant (A) for the given subject. We wish to recover $q(s, m)$ from the data, which we treat as described next.

Treatment of data

In our experiments, the roughness magnitudes for a given plate and condition are averaged over replications to provide a stable value of mean roughness. The resulting average is normalized by dividing by the subject's mean across conditions and stimuli (i.e., spacing values), then rescaled by multiplying by the grand mean over subjects, conditions, and stimuli. Then logs are taken of the normalized values. The logs are averaged across subjects, producing an empirical function relating mean roughness magnitude to interelement spacing.

Proof Notation

s denotes a stimulus plate's spacing value.

i denotes a subject.

m denotes a condition (e.g., modality).

$\langle \text{stuff} \rangle_{x,y,z}$ is a mean of the expression "stuff" over x, y, and z.

q(s, m) is a quadratic function of s that generates perceived magnitudes in condition m.

a, b, and c are the quadratic, linear, and constant terms of q.

X_i is the response of the i^{th} subject. It is denoted $X_i(s,m)$ because it depends on stimulus and condition. The normalized response (defined below) is denoted $X'_i(s,m)$.

A_i is the proportionality constant for the i^{th} subject.

Proof

(1) For a particular condition m and spacing value s there is a quadratic function q(s, m) as follows.

$$q(s, m) = a(m) * (\log s)^2 + b(m) * (\log s) + c(m)$$

2) The i^{th} subject's response is the exponentiation of the output of q(s, m), multiplied by the subject's proportionality constant, A_i .

$$X_i(s, m) = A_i * e^{q(s,m)}$$

3) The i^{th} subject's response is normalized by dividing by the subject's mean across conditions and spacing $\langle X_i(s,m) \rangle_{s,m}$ and multiplying by the grand mean $\langle X_i(s,m) \rangle_{i,s,m}$.

$$X'_i(s, m) = \{ X_i(s, m) / \langle X_i(s, m) \rangle_{s,m} \} * \langle X_i(s, m) \rangle_{i,s,m}$$

4) Next we take the log of the normalized response, and the multiplicative relations in Equation 3 become additive.

$$\log X'_i(s, m) = \log X_i(s, m) - \log \langle X_i(s, m) \rangle_{s,m} + \log \langle X_i(s, m) \rangle_{i,s,m}$$

Note that the term $\langle X_i(s,m) \rangle_{s,m}$ is a mean that depends only on i, and $\langle X_i(s,m) \rangle_{i,s,m}$ is the grand mean, which is a scaling constant. When we consider a given subject, we can ignore these two terms, as they are constants. We will designate the constants as Y_i and Z , respectively, in order to simplify subsequent formulas; that is, Y_i is the i^{th} subject's overall mean, and Z is the grand mean. The new expression is then:

$$\log X'_i(s, m) = \log X_i(s, m) - \log Y_i + \log Z$$

5) Substituting for $X_i(s,m)$ in the above expression, using Equation 2, expresses the normalized, logged subject's response in terms of the underlying quadratic function q .

$$\begin{aligned}\log X'_i(s,m) &= \log X_i(s,m) - \log Y_i + \log Z \\ &= \log\{A_i * e^{(q(s,m))}\} - \log Y_i + \log Z \\ &= \log A_i + q(s,m) - \log Y_i + \log Z\end{aligned}$$

6) Next we take the mean over subjects of the normalized, logged responses as represented by Equation 5. Note that as $q(s,m)$ does not depend on i , it remains unchanged, as does Z .

$$\langle \log X'_i(s,m) \rangle_i = \langle \log A_i \rangle_i + q(s,m) - \langle \log Y_i \rangle_i + \log Z$$

Also note that the three terms other than $q(s,m)$ are constants, because the expressions depending on i have now been averaged over i . Rearranging and combining terms yields:

$$\langle \log X'_i(s,m) \rangle_i = q(s,m) + \{\langle \log A_i \rangle_i - \langle \log Y_i \rangle_i + \log Z\}$$

7) Next substitute for $q(s,m)$ as per Equation 1. This expression is the mean over subjects for a given stimulus (i.e., spacing value) and condition.

$$\langle \log X'_i(s,m) \rangle_i = a(m) * (\log s)^2 + b(m) * (\log s) + c(m) + \{\langle \log A_i \rangle_i - \langle \log Y_i \rangle_i + \log Z\}$$

Parameters recovered by fitting a quadratic to the normalized, logged magnitudes, as a function of log spacing:

In our data analysis, the mean $\log X'$ values from Equation 7 are fit by a quadratic of $\log s$ for a given condition, m . We consider here what parameters of the underlying function, q , which generated the initial data, are preserved by the steps in the analysis. From Equation 7, the quadratic will recover $a(m)$ and $b(m)$ absolutely, because they are coefficients involving $\log s$. But the quadratic's constant term includes not only $c(m)$ but the other constants within the brackets. Thus the constant represents the sum of four components: the constant term of the underlying function, the average of the subjects' log proportionality constants, the average of the subjects' log means, and the log of the grand mean. Note, however, that the last two terms of the constant do not vary with the condition, m . In other words, the $c(m)$ term is recovered within a constant that is invariant across conditions, which preserves the additive relations between conditions with respect to $c(m)$.

What is recovered under other assumptions or normalization procedures. If one were to assume different proportionality constants across conditions for a given subject, the A_i term in the constant would become $A_i(m)$. Hence the quadratic and linear components would be preserved, but now the constant would vary with the condition, and the preservation of additive relations among constant terms would not be guaranteed.

Similarly, if we were to change our normalization so that we divided by the subject's condition mean, then $\log \langle X_i(s,m) \rangle_{s,m}$ would become $\log \langle X_i(s,m) \rangle_s$, which

would depend on i and m . Thus what has been designated as Y_i would become $Y_i(m)$, and this component of the constant would vary with m . Again, the quadratic and linear components would be preserved, but now the constant would vary with the condition, and the preservation of additive relations among constant terms would not be guaranteed..