Discussion Questions for
HUDK 4027 Development of Mathematical Thinking
January 18 to February 22, 2012
*The Number Sense: How the Mind Creates Mathematics, Revised & Expanded Ed.* (Dehaene)

1/25/12: Chapter 1, Talented and Gifted Animals

1. What are the implications of the Clever Hans story for whether researchers should conduct their own studies (as opposed to letting research assistants who do not know the hypotheses of the study conduct them)? How have animal cognition researchers gotten around the problem of subtle, unconscious biases among experimenters?

2. What are the implications of the Clever Hans story for interpreting difficult to understand modern claims, such as those regarding ESP (extra sensory perception), seers such as Jean Dixon, and 1- and 2-year-olds' learning to read and do math from being presented flashcards?

3. Are you surprised that animals such as rats and pigeons can do simple numerical operations? Why might evolution have prepared these animals to be able to process numbers in relatively precise ways?

4. Why might animals code approximations of numbers rather than exact numbers? Why might the amount of variation of rats' number of bar presses be greater for larger numbers than for smaller ones (p. 9)?

5. What, if anything, do you think it means that in Meck and Church (1983), the rats' midpoint of bar presses was 4 rather than 5, when the original numbers that they needed to generate were 2 and 8 (pp. 10-11)?

6. Does the experiment described on p. 14 convince you that the chimps are adding the fractions? What other possible interpretations would need to be ruled out before you would accept this conclusion?

7. What is the accumulator model? What are its advantages and disadvantages as a model of arithmetic for humans and other animals?

8. What are distance and magnitude effects? Why do you think they're so widespread among animals (including people)?

9. Does Dehaene's neural model (pp. 20-23) imply that animals have neurons dedicated to detecting, for example, the numbers 45 and 50? If not, how do animals make this discrimination, which Dehaene earlier indicated that they can make?

10. Dehaene describes the chimpanzee Ai as having learned to add pairs of the first 9 numbers with 95% accuracy (p. 36) and indicates that analyses of the chimp's response times suggest that he uses serial counting for all except the first few numbers. What are

the implications of animals being able to learn to do simple numerical activities but taking a great amount of time and training to do so?

11. Describe each stage of the Boysen experiment with Sheba. At what point, if any, would you say that Sheba demonstrated an understanding of numbers?

12. What were the implications of Sheba understanding immediately that the symbol for representing 2+2 would be 4, as opposed to her requiring training before she showed similar understanding?

1/25/12: Chapter 2, Babies Who Count

13. Why do you think children fail number conservation tasks, when they have other types of understanding about numbers? Why do they fail class inclusion tasks (six tulips, two roses, more tulips or flowers)? How can we reconcile infants' and toddlers' competence in some aspects of numerical understanding with older children's lack of competence in other aspects?

14. Were the procedures used by Mehler and Bever (1967) (pp. 33-34) and by McGarrigle and Donaldson (1974) really conservation problems? Why or why not?

15. If 3- and 4-year-olds interpret the second question on each conservation trial in the Mehler and Bever (1967) study as meaning that the experimenter wants a different answer, why would both 2- and 5-year-olds not interpret it that way?

16. Why would infants attend to three objects when they hear three sounds (p. 40)? Do they have good reason to do so? Can you think of reasons other than their matching the number of sights and sounds why they might look more at three objects when hearing three sounds and two objects when hearing two sounds?

17. Should the long looking time in Wynn's experiments and other habituation paradigm studies on the dishabituation trial be viewed as indicative of surprise? Can you think of other reasons why babies might look for a long time at certain displays even if they were not surprised?

18. Do you find the interpretation in the last paragraph on p. 44 of why babies attend to number but not identity to be convincing? Why or why not?

19. Dehaene suggests that babies assume that a truck can turn into a ball but not that one object could become two objects (p. 48). Is this reasoning sound; why or why not?

20. Dehaene bases many of his interpretations in evolutionary theory. Would it do babies much good for survival to recognize the number of objects in very small sets if they did not know which of two sets was more numerous? Why might evolution provide such an ability if it wouldn't do babies much good?

2/01/12: Chapter 3, The Adult Number Line

21. Do Dehaene's examples (p. 54) support his claim that "most if not all civilizations stop using this system beyond the number 3?" Does the exact point at which the initial system is abandoned matter for our interpretation of the phenomenon (why does Dehaene think it does)?

22. Can you think of reasons why there might be a discontinuity between numbers 3 and 5 in our ability to subitize?

23. Does the evidence from brain damaged patients (p. 59) favor the parallel preattentive account over the counting hypothesis? Why or why not?

24. Does it surprise you that human number abilities resemble those of rats and pigeons in many ways (p. 61)? Can you generate a hypothesis regarding how humans and animals are similar in number processing and how they differ?

25. Why do you think that the extensive training that Dehaene provided on whether 1, 4, 6, and 9 were greater or smaller than 5 failed to change the distance effect for those numbers (p. 63)? What does this tell us about numerical knowledge?

26. Why do currencies in all countries have more coins and bills corresponding to small values than large ones? Might this be related to the reasons why people and other animals seem to have a logarithmic ruler for representing quantity?

27. What does it mean to say, "Understanding numbers, then, occurs as a reflex" (p. 66)?

28. If numerical magnitude is understood as a reflex, and we are upset by discontinuities as in the train platform and calendar examples, why is $3.99 perceived as noticeably less than $4.00 (p. 68)? Doesn't the general logic of this section argue in the opposite direction?

29. Why do numbers seem to be represented spatially? Is it just because we've all seen number lines in school (or do we have number lines because people tend to represent numbers spatially)? What evidence would help you answer the question?

30. What is the SNARC effect? In what way does it provide evidence for spatial representations of numbers?

31. What does it tell us that 4 and 5 are associated with the right side when only numbers 1-5 are used and with the left side when only numbers 4-9 are used?

32. "If we did not already possess some internal non-verbal representation of the quantitiy 'eight,' we would probably be unable to attribute a meaning to the digit 8" (p. 75). Do you agree with this statement? Do we also have a non-verbal representation of the number "53"? How about "553"? If not, are these numbers meaningless?

33. What defines rational numbers? Irrational numbers? Complex numbers?

2/01/12: Chapter 4, The Language of Numbers

34. If animals' numerical representations are based on the accumulator model, and human language is needed to express other types of numbers such as fractions, how is it that animals can represent fractions, as described in Chapter 1?

35. Most languages have systematic, hierarchical terms for representing numbers; for example, to represent "426", we concatenate a number of hundreds, a number of 10's, and a number of 1's. Not all languages do so, however. What do you think determines whether a language does or does not adopt a hierarchical system for representing numbers?

36. Why did base 10 become the most common system in widely dispersed societies? Why not base 2, 5, or 20?

37. Why was the invention of 0 so important?

38. Why is it advantageous to have number symbols that are visually unrelated to the numbers that they represent?

39. If the East Asian system of representing numbers is more efficient (pp. 89-90), as it seems to be, why don't we in the U. S. just adopt an English equivalent?

40. If children could vote, would they choose the Chinese model of number names?

41. How much good would the principle of contrast do the hypothetical baby Charlie in learning the meaning of 3 (p. 94)? Can you think of other hypotheses that a baby might develop other than that 3 refers to the number of puppies (and that English language input that they hear)?

42. Why does "a mile or three" never sound right (p. 96)? What rules does Dehaene suggest govern the degree of precision that phrases like this convey? Do you agree that those rules govern our use of approximate numbers?

43. Do you find it surprising that the numbers 1-3 appear so much more often than other numbers? Why might this occur?

44. Do you think Dehaene's summary of the history of oral and written numeral systems is useful? Why or why not? What else should it have; what should be left out?

2/08/12: Chapter 5, Small Heads for Big Calculations

45. Does Dehaene's claim that counting is somewhat innate, as well as somewhat learned, surprise you? What evidence does he cite to support his claim, and how compelling do you find the evidence? Why do you find it persuasive or unpersuasive?

46. Why do young children like to count objects, if they do not know the purpose of counting (p. 106-107)?

47. If children possess the accumulator model from birth, as Dehaene suggests, why does it take them until the end of their fourth year to understand the purpose of counting? Is it plausible that it only takes them 10 or 20 counts to understand it, given that counting is a very common activity for many 3-year-olds?

48. What role do fingers serve in children's learning of arithmetic?

49. Dehaene claims on p. 110 that "There was only one way out of this conundrum." Is he right; can you think of alternatives? (He's wrong on this one; see if you can figure out what actually happens.)

50. Do some of you use strategies other than retrieval on problems such as 6X9 and 7X8? What strategies do you use? Does it surprise you that about half of college students do this?

51. Is the comparison to multiplication of the arbitrary facts on p. 112-113 a persuasive one? What is the difference between learning multiplication facts and learning the logical statements?

52. When you read the example on p. 114, did you do what Dehaene said you would? What does his example tell us about arithmetic?

53. Toward the bottom of p. 117, Dehaene provides an example, and writes, "suggesting that in parallel to calculating the exact result, our brain also computes a coarse estimate of its size." What general lesson does this example, together with his general emphasis on the verbal nature of arithmetic, have for understanding how the brain solves problems?

54. What does it mean to say that on 7X9=20, parity is violated (p. 117)? Are there parity rules for all four arithmetic operations? If so, what are they? Did you know these rules before you thought about them in connection with this question and the comment in the text?

55. Why don't textbooks typically spell out the long subtraction algorithm (p. 118)? Would arithmetic learning be enhanced if they did so?

56. Does the frequency of subtraction bugs indicate that, "the very occurrence of such absurd errors suggests that the child's brain registers and executes most calculation algorithms without caring much about their meaning?" (p. 119)? What alternative explanations can you generate?

57. Dehaene argues that relying more on calculators, rather than practice with arithmetic facts, would lead to better understanding of mathematical concepts. What is his logic? What logic would lead to the opposite prediction? Do you think that children today who are good at calculation become adults who are generally good at mathematics? Why?

58. From your experience is it true that "The vast majority of adults never perform a multidigit calculation without resorting to electronics"? If you sometimes calculate in your head or on paper, why do you do it?

59. How would you teach children math in ways that would avoid the errors on p. 122?

60. Why does adding the numerators and adding the denominators work in the example of Michael Jordan's shooting but not in the pie example (p. 126)? How would you teach children the difference between the two cases? Would you follow Dehaene's advice to tell children that when discussing fractions addition, think of pies rather than scoring averages?

2/08/12: Chapter 6, Geniuses and Prodigies

61. Dehaene emphasizes similarities between mathematical geniuses and idiot savants who are good at identifying prime numbers and related feats. Do you see the unusual abilities of the two groups as being related? What kind of data would be crucial for deciding whether they are in fact related?

62. Why do multiples of 9 have ones digits that descend by one number for each multiple (9, 18, 27, 36…) (P. 133)?

63. Dehaene (p. 136) notes that Einstein, among others, claims that his great insights occurred without the involvement of language. Do you think that this is generally true for the rest of us, who have insights from time to time but are not among the great geniuses of history?

64. Claims were made in the 19$^{th}$ Century that the brains of women were smaller than those of men, and that women were therefore less intelligent. Subsequent research has shown that the brains of women are indeed smaller on average than those of men, but that there are no differences in average intelligence between men and women. Why, then, might women's brains typically be smaller?

65. As Dehaene notes on p. 141, the larger sizes of certain brain areas relevant to musical performance in musicians is as likely due to their experience playing their instrument as to any innate difference in their brains. Design an experiment to determine if there are innate differences in the brains of musicians independent of their musical experience. Don't worry about the expense of the experiment.

66. The gender differences in mathematical test scores that Dehaene describes on pp. 143-146 have changed somewhat in the years since the studies that he describes were done. Which phenomena that he describes would you guess have changed (and why), and which would you expect to have stayed the same?

67. Within the testosterone explanation of sex differences in mathematical giftedness that Dehaene advances on p. 146, why might the number of men at the superior level of math achievement be higher than the number of women, even if average math achievement is the same for men and women?

68. The experiments regarding expert memorizers of numbers that Dehaene describes on p. 148 took advantage of certain ways that the students spontaneously remembered the numbers. Can you anticipate what some of them might be? (Hint: One of the memory experts was a runner.)

69. Great mental calculators almost always calculate from left to right. Why do you think they do this?

70. It is plausible that children learn through experience with the particular numbers that there are some numbers, such as 12, that can be divided into equal size groups, and others, such as 13, that cannot (p. 153)? But how does anyone learn this (except from instruction) for large and uncommon numbers such as 389 or for roots of 179,859,375?

71. In his conclusion (pp. 155-156), Dehaene says that biology probably plays a part in great mathematical achievement, but then says that biology does "not weigh much compared to the powers of learning, fueled by a passion for numbers." Is it purely a matter of a supportive environment whether a person develops a passion for numbers? Might biology play a role there too?

<u>2/15/12: Chapter 7, Losing Number Sense</u>

72. What are the ideas of "dissociation" and of "double dissociation"? Why is double dissociation a particularly valued type of evidence in the study of brain-damaged patients? What kinds of hypotheses does it allow us to rule out?

73. What parts of Dehaene's summary of Mr. N's condition, a condition in which Mr. N could approximate but lacked exact knowledge, do and do not fit his observations of Mr. N?

74. What is the logic of split-brain studies; what have they told us about processing of numbers in each hemisphere? Why might findings from them not be generalizable to people with intact brains?

75. Do people need to understand math to answer problems such as 4 minus 3? Is Mr. M's problem not understanding the magnitudes of the numbers 4 and 3, or does it lie elsewhere (and if so, where)?

76. Why might the inferior parietal lobe be specialized for mathematical processing, in particular for "the number sense"? What principles of brain organization are implicit in Dehaene's arguments for the view that the inferior parietal lobe is the prime location of the number sense?

77. If Mr. M doesn't have a sense of magnitude, how can he estimate "the duration of Columbus' trip to the New World or the distance from Marseilles to Paris" (p. 178)?

78. Does the specificity of processing pathways for reading and understanding different types of material surprise you (pp. 179-181)? What are the implications of such extreme specificity of content knowledge for the future of cognitive neuroscience?

79. Some authorities have argued that the prefrontal cortex is particularly crucial in making human beings human. Do Dehaene's descriptions of prefrontal functioning in math support or contradict this view? How so?

80. What is cortical plasticity? Why is it so important for learning mathematics and other "unnatural" activities?

2/15/12: Chapter 8, The Computing Brain

81. What are the advantages of brain imaging technologies over studying the thinking of brain damaged patients? Do we gain any valuable information from studying brain damaged patients that we do not gain from neuro-imaging studies?

82. Why does Lennox's experiment seem prescient from a modern perspective? Is it true that, "the intense effort required by mental calculation had to be responsible for the increase (in oxygen) observed in the experimental group"? (Or is Dehaene's statement a couple of pages later more accurate that, "The increase in oxygen content that Lennox observed may have had little to do with mental calculation. It could simply have been due to the intense perceptual and motor activity required to scrutinize a sheet filled with mathematical signs…"

83. How does PET work? Why is blood flow in the brain relevant to cognitive activity?

84. What was Dehaene's objection to the Roland and Friberg experiment? How does his interpretation differ from theirs? Could some future scientist have the same objection to Dehaene's interpretation of the Roland and Friberg findings?

85. Why did Dehaene anticipate inferior parietal activation on the magnitude comparison task but not on the multiplication task (p. 202)? What are the implications of the reverse actually being the case (more IPS activity on the multiplication task)?

86. Why is it important to present a specific event within event-related potential research, rather than just watching the brain functioning (p. 206)?

87. What is the main limitation of PET technology? What is the main problem with EEG technology? What properties would an ideal imaging technology possess?

88. What does single cell recording add to the information that can be gained from PET and other imaging technologies?

2/22/12: Chapter 9: What Is a Number?

89. Why was the metaphor of the brain as a computer appealing? What are the arguments against it? Does the "brain as an information processing device" overcome the problems with the "brain as computer" metaphor? Explain your views.

90. What do you think of Dehaene's educational recommendation starting with "One should first arouse their curiosity..." and the rest of the first full paragraph on p. 224?

91. On p. 231, Einstein is quoted as asking, "How is it possible that mathematics, a product of human thought that is independent of experience, fits so excellently the objects of physical reality?" The question stands even if we reject the idea that mathematics is independent of experience. That leaves us with the question of why so many aspects of nature can be described by relatively simple mathematical laws. Any ideas why?

92. How does the variation and selection perspective explain the "hand in glove fit" of mathematics to physical reality (p. 232)? Do you find this explanation compelling? Why or why not?

93. Dehaene asks at the end of the chapter (p. 233), "Is the universe really "written in mathematical language" as Galileo contended?" He answers his question, "I am inclined to think instead that this is the only language with which we can try to read it." What is Dehaene saying, and do you agree with his view?

2/22/12: Chapter 10: The Number Sense, Fifteen Years Later

94. Dehane's central construct, now and 15 years ago, is number sense. What exactly is number sense, and what aspects of numerical knowledge does it not include?

95. Number, space, and time are closely linked. The relation is present in infancy, before children learn language, but it is also reflected in language. Can you think of some ways that language reflects these close relations?

96. How do space and number judgments interact in patients with spatial neglect (p. 244)?

97. Can you think of applications of the findings on eye movements at the bottom of 245 and top of 246 to improving your ability to play poker?

98. Nieder and his colleagues have found neurons tuned to numbers in the 20's and 30's (and from a talk I heard him give recently, as high as 64). How could a monkey use such information?

99. What is the difference between the response characteristics of "number neurons" found in the VIP and LIP areas? Why are both necessary?

100. Do you find the habituation/dishabituation experiment (Piazza, et al., 2004) convincing that the IPS is the key area for numerical magnitude representations (pp. 253-254)? Why or why not?

101. Why might infants and 4-year-olds only represent number in the right IPS, whereas adults and older children represent it bilaterally (in both sides of the cortex)?

102. Does it surprise you that the Mundurucu are almost as accurate as French adults in the ratios of numbers that they can discriminate (p. 262)? What does this say about the approximate number system?

103. The Munduruku's number line estimation shows both influences of the approximate number system and influences of experience (or lack of such) with counting systems pp. 264-266). Describe each type of influence.

104. Why would numerical discrimination of sets become increasingly precise over time in cultures where children attend school?

105. Do you think that Dehaene's account of how approximate number representations give rise to precise representations is plausible? If not, why not? Either way, can you think of other ways in which neural representations of precise quantities would arise and be represented in the brain?

106. Why is the precision of children's approximate number representations related to their math achievement test scores, which involve the types of exact number operations (mainly arithmetic) that are taught in school( p. 273)?

107. Is it true that "The fact that here is a category of children with normal intelligence and schooling, but a disproportionate deficit in arithmetic, disproves the notion that education always involves domain general learning mechanisms"? (p. 275) What other explanations can you produce?

108. Why do you think the number board game described on p. 277 works as well as it does? What are its specific features that lead to the large effects on numerical understanding that it produces?