Chinese Children Excel on Novel Mathematics Problems Even Before Elementary School

Robert S. Siegler and Yan Mu
Carnegie Mellon University

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Address correspondence to Robert S. Siegler, Department of Psychology, Carnegie Mellon University, Pittsburgh, PA 15213; phone: 412-268-2809; e-mail: rs7k@andrew.cmu.edu.

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Abstract

Preschoolers in China were found to have greater numerical knowledge than preschoolers in the U. S., not only of arithmetic problems, which Chinese parents present more often than U. S. parents, but also of unfamiliar number line estimation problems, which were novel to children in both countries. The Chinese preschoolers’ number line estimates were comparable to those of U. S. children one to two years more advanced in school. Individual differences in arithmetic and number line estimation performance were positively correlated within each country. These results indicate that differences between Chinese and U. S. children on both practiced and unpracticed mathematical tasks are substantial even before the children begin elementary school.
Chinese Children Excel on Novel Mathematics Problems Even Before Elementary School

Children in China, Japan, and other East Asian countries outperform American age peers on numerous mathematical tasks: counting, arithmetic, algebra, geometry, etc. (Ginsburg, et al, 1997; Stevenson, Chen, & Lee, 1993). This “learning gap,” evident as early as kindergarten, persists throughout elementary and high school and probably beyond (Stevenson & Stigler, 1992).

Most explanations of the learning gap have focused on schooling. Children in East Asia spend more time on math in classrooms, devote more time to doing math homework after school, and encounter more challenging math problems in each grade (Chen & Stevenson, 1989; Stigler & Hiebert, 1999). East Asian teachers more deeply understand fundamental mathematics concepts and use more focused pedagogical practices (Ma, 1999), provide more substantive explanations of procedures (Perry, 2000), and more often promote multiple solutions to a given problem (Geary, 1994).

Differing cultural emphases also seem to contribute to the learning gap. Parents in China place greater emphasis on the importance of mathematics, and are more involved in their children’s math learning, than parents in the U.S. (Huntsinger et al, 1997; Zhou et al, 2006). Even before Chinese children enter school, their counting and adding numbers with sums of 10 or less is superior (Geary et al, 1993; Geary et al, 1996).

The basic question that motivated the present study was whether the superior mathematical knowledge of East Asian preschoolers is limited to skills that are taught directly by parents or whether the superiority is more general. The skills on which cross-national differences among preschoolers have been documented – counting and adding –
are ones for which Chinese parents provide substantial practice before their children begin elementary school (Zhou et al., 2006). However, Chinese parents do not provide instruction on other, less routine mathematical tasks, such as numerical estimation (Zhou et al., 2006). This raises the issue of whether the learning gap among young children is merely a superficial result of rote learning, or whether differences in mathematical knowledge before Chinese and U. S. children begin elementary school include ability to solve novel problems.

In the present study, we examined Chinese and American preschoolers’ knowledge of a practiced and an unpracticed task. The practiced task was arithmetic (addition of single digit numbers). The unpracticed task was number line estimation (Siegler & Opfer, 2003). On this task, children were presented number lines with 0 at one end, 100 at the other, and nothing in between. The experimenter asked children to locate the position of a number on a number line; then gave them a fresh, identical number line and asked them to locate a different number on it; and so on. The success of the best fitting logarithmic, linear, and exponential functions in accounting for each child’s estimates of all presented numbers was then compared.

Optimal performance on the number line task entails the best fitting linear function accounting for 100% of the variance in the estimates, with a slope of 1.00. American adults’ performance closely approximates this ideal (Siegler & Opfer, 2003). In contrast, American children exhibit a consistent age-related progression from the use of logarithmic estimation patterns to the use of linear ones. On 0-100 number lines, the large majority of kindergartners generate estimates best fit by a logarithmic function, the large majority of second graders generate estimates best fit by a linear function, and about half
of first graders’ patterns are best fit by each function (Geary et al., 2007; Siegler & Booth, 2004). As shown in Figure 1, this means that kindergartners’ and many first graders’ estimates of numerical magnitude rise too quickly at the low end of the scale and are too compressed at the high end. The developmental pattern repeats itself between second and fourth grade in the 0-1,000 range (Booth & Siegler, 2006).

Linear representations of numerical magnitudes appear to be crucial to children’s mathematics performance and learning. The linearity of individual children’s number line estimates correlates strongly with performance on numerous other mathematical tasks (Booth & Siegler, 2006; Laski & Siegler, 2007). Linearity of estimates also correlates substantially with overall math achievement test scores from kindergarten through fourth grade (Booth & Siegler, 2006; Siegler & Booth, 2004). Perhaps most striking, providing randomly selected children with visual representations of the magnitudes of addends and sums along a number line improves children’s learning of answers to arithmetic problems (Booth & Siegler, in press).

The current study is the first to examine Chinese kindergarteners’ proficiency on the number line estimation task. We hypothesized that Chinese preschoolers’ greater practice at arithmetic and counting would improve their understanding of numerical magnitudes, even on unpracticed tasks such as number line estimation. Consider how the most common early addition strategy, counting fingers, could contribute to their knowledge of numerical magnitudes. When counting fingers to solve addition problems, the larger the sum, the more fingers children put up, the more fingers they see, the more numbers they count, and the more time they take to reach the sum. For example, solving 4+4 by counting fingers will require putting up and seeing twice as many fingers, saying
twice as many numbers, and taking roughly twice as much time as solving 2+2. These
kinesthetic, visual, verbal, and temporal cues provide broad-based support for a sense of
numerical magnitudes. Thus, the greater arithmetic and counting experience of Chinese
children was expected to produce superior number line estimation, despite neither
Chinese nor U. S. children having experience with that task.

The same logic implies that within each society, individual differences in arithmetic
and number line estimation would correlate positively. Children who are more
experienced and skilled at arithmetic than peers within their society should be better at
the unpracticed number line task as well. Thus, the three main predictions of the
experiment were:

1. Chinese preschoolers’ number line estimates will be more accurate, more linear,
   and have slopes closer to 1.00 than those of American peers.
2. Chinese preschoolers’ arithmetic skill will exceed that of American peers.
3. Individual children’s number line proficiency will correlate positively with their
   arithmetic skill in both Chinese and American samples.

Method

Participants

The 29 Chinese children (mean age = 68 months, range = 63-73 months; 11 girls) were recruited from a preschool affiliated with Tianjin Normal University. The 24
American children (mean age = 67 months, range = 63-75 months, 12 girls, 88%
Caucasian, 12% Asian) were recruited from a preschool affiliated with Carnegie Mellon
University. Conversations with school personnel at the two sites indicated that the socio-
economic status of the Chinese children was no higher, and perhaps lower, than that of
the American children, and that the preschool in China was less prestigious relative to other Chinese preschools than the one in America was relative to other American preschools. The principals’ and teachers’ descriptions were consistent with the knowledge of the second author, who had lived in both cities and was familiar with both preschools.

Procedure

Problems and instructions on the number line task were like those in previous studies (Siegler & Booth, 2004). Among the 26 numbers whose positions children estimated, 4 were from each of the first three decades and 2 from each subsequent decade. The reason for oversampling the first three decades was to better discriminate between linear and logarithmic estimation patterns. The numbers -- 3, 4, 6, 8, 12, 14, 17, 18, 21, 24, 25, 29, 33, 39, 42, 48, 52, 57, 61, 64, 72, 79, 81, 84, 90, and 96 -- were ordered randomly.

The addition task was one used previously by Geary et al. (1996) to compare the arithmetic knowledge of Chinese and U.S. kindergarteners. It consisted of 70 addition problems with sums between 2 and 10. Children were given one minute to answer as many items as possible. The two tasks were presented in random order.

Results

Number Line Task

To examine the accuracy of children’s estimates, we computed their percent absolute error (PAE) using the formula: $\text{PAE} = \frac{|\text{Estimate} – \text{Estimated Quantity}|}{\text{Scale of Estimates}}$. Thus, if a child was presented the number 85 and estimated its location at
the point that corresponded to 75, the child’s PAE on that trial would be 10% (\(| 75 - 85 | /100\)).

The American children’s mean PAE was 22% (SD=8.3). The Chinese children were considerably more accurate, mean PAE =15% (SD=7.7), \(t(51) = 3.05, p < .01, d = .82, p_{rep} = .977\). To place these findings in context, the Chinese preschoolers’ mean PAE was between that of American first and second graders in previous studies (e.g., Booth & Siegler, 2004).

We next examined the linearity of children’s median estimates for each number. As in previous studies, American preschoolers’ median number line estimates of each number’s magnitude were better fit by a logarithmic function than by a linear function, \(R^2_{\text{log}} = .90 > R^2_{\text{lin}} = .72\). Unlike the findings with any previous American sample, the Chinese preschoolers’ median estimates were better fit by a linear function than by a logarithmic function, \(R^2_{\text{lin}} = .95 > R^2_{\text{log}} = .86\). As Figure 2 shows, the Chinese preschoolers generated a highly linear pattern of estimates.

Analyses of individual children’s estimates told a similar story. For Chinese children, the best fitting linear function accounted for greater variance in individual children’s estimates than the best fitting logarithmic function: means of 63% and 58%, \(t(28) = 2.83, p < .01, d = .54, p_{rep} = .959\). In contrast, among American children, the best fitting logarithmic function accounted for greater variance in individual children’s estimates than the best fitting linear function: means of 53% and 43%, \(t(23) = 3.89, p < .01, d = .65, p_{rep} = .992\). The mean variance accounted for by the linear function was greater in the Chinese sample than in the American sample, 63% versus 43%, \(t(51) = 2.22, p < .05, d = .65, p_{rep} = .908\). More Chinese than American students’ estimation
patterns were better fit by a linear function than by a logarithmic one, 62% vs. 17%, Chi Square (1) = 11.15, p < 0.01.

The slopes of the best fitting linear function also tended to be closer to 1.00 for the Chinese than for the American children, .53 vs. .39, $t(51) = 1.85, p < .10, d = .50, p_{rep} = .85$. Thus, analyses of individual children’s accuracy, linearity, and slopes converged on the conclusion that the Chinese preschoolers’ performance on the novel number line task was considerably more advanced than that of American preschoolers.

Arithmetic Performance

Chinese preschoolers also correctly answered more addition problems than did their American peers, 8.4 versus 5.3 (SD’s = 3.8 and 4.2), $t(51) = 2.91, p < .01, d = .88, p_{rep} = .97$.

Relation Between Number Line Estimation and Addition

Within both U.S. and Chinese samples, individual differences in number of correct addition answers were related to individual differences in quality of number line estimates, both when estimation quality was indexed by PAE, $r_{US} (22) = -.64, p < .01; r_{China} (27) = -.38, p < .05$ and when estimation quality was indexed by linearity, $r_{US} (22) = .60, p < .01; r_{China} (27) = .40, p < .05$.

Discussion

Results of this study were consistent with its three main hypotheses. Chinese preschoolers’ number line estimates were more advanced than those of American peers. Chinese preschoolers’ arithmetic performance was also more advanced. Among preschoolers in both countries, individual differences in number line estimation and arithmetic proficiency were positively correlated. Thus, even before they enter
elementary school, Chinese children’s mathematical knowledge is superior on tasks that parents do not present as well as on ones that they do, and individual differences in performance on familiar and unfamiliar numerical tasks are related within both societies.

The literature on intersensory redundancy and learning provides a useful context for thinking about the present findings. Providing multiple, synchronous, redundant cues promotes many types of learning, including numerical learning. For example, Jordan, Suanda, and Brannon (2008) found that providing redundant visual and auditory cues to number allowed 6-month-olds to make finer discriminations among sets with varying ratios of dots than was the case with visual or auditory cues alone. Similarly, intersensory redundancy between vocalizations and object motions facilitates infants’ learning of speech-object relations (Gorgate & Bahrick, 1998). Our predictions that Chinese children’s numerical estimation would be more advanced, and that individual differences in arithmetic and number line estimation would be correlated, were based on a similar logic. The practiced activities of counting fingers and other objects to solve arithmetic problems and to determine set sizes convey redundant kinesthetic, visual, auditory and temporal information about numerical magnitudes. Practice in adding and counting is surely not the only source of differences between Chinese and U. S. children’s numerical knowledge, but it may well be one source.

A similar analysis gave rise to a previous prediction that playing a numerical board game akin to Chutes and Ladders would improve preschoolers’ knowledge of numerical magnitudes. Such games provide the same type of redundant cues to numerical magnitude as does counting fingers to solve arithmetic problems. The greater the number in a square of the board game, the greater the number of movements to reach it, the
greater the number of words the child has said and heard, the further the distance of the
token from the origin, and the longer the time from the beginning of the game. Consistent
with this analysis, preschoolers from low-income backgrounds who were randomly
assigned to play a linear number board game improved their number line estimation and
magnitude comparison performance more than children who played a similar game on a
board with different colors rather than different numbers in the squares (Ramani &
Siegler, 2008; Siegler & Ramani, in press). As these and the present study illustrate,
analyzing everyday activities in terms of the cues they provide for inducing desired
concepts may advance understanding of cross-cultural, individual, and social class
differences in knowledge and learning.
References


Figure Caption

1. U. S. kindergartners’ median estimates increase logarithmically with numerical magnitude, whereas U. S. second graders’ estimates increase linearly (data from Siegler & Booth, 2004, Experiment 1).

2. U. S. preschoolers’ median estimates increase logarithmically with numerical magnitude, whereas Chinese preschoolers’ estimates increase linearly.
Median Estimates: Kindergarten

Estimated Magnitude

Median Estimates: Second Grade

Estimated Magnitude

\[ y = 14.51 \ln(x) + 8.74 \]
\[ R^2 = 0.75 \]

\[ y = 0.64 x + 19.46 \]
\[ R^2 = 0.95 \]
Median Estimates: US

Estimated Magnitude

\[ y = 15.87 \ln(x) - 2.39 \]
\[ R^2 = 0.90 \]

Median Estimates: China

Estimated Magnitude

\[ y = 0.63x + 17.77 \]
\[ R^2 = 0.95 \]