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Playing Linear Number Board Games—But Not Circular Ones—Improves Low-Income Preschoolers' Numerical Understanding

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A theoretical analysis of the development of numerical representations indicated that playing linear number board games should enhance preschoolers' numerical knowledge and ability to acquire new numerical knowledge. The effect on knowledge of numerical magnitudes was predicted to be larger when the game was played with a linear board than with a circular board because of a more direct mapping between the linear board and the desired mental representation. As predicted, playing the linear board game for roughly 1 hr increased low-income preschoolers' proficiency on the 2 tasks that directly measured understanding of numerical magnitudes—numerical magnitude comparison and number line estimation—more than playing the game with a circular board or engaging in other numerical activities. Also as predicted, children who had played the linear number board game generated more correct answers and better quality errors in response to subsequent training on arithmetic problems, a task hypothesized to be influenced by knowledge of numerical magnitudes. Thus, playing linear number board games not only increases preschoolers' numerical knowledge but also helps them learn from future numerical experiences.

Keywords: math, number, preschoolers, low income, arithmetic

Among the most serious educational challenges facing the United States is the large discrepancy in academic performance between children from different economic backgrounds. Children from impoverished backgrounds achieve at a much lower level than other students throughout the course of schooling (e.g., Alexander & Entwisle, 1988; Geary, 1994, 2006). One important reason is that these children start school with far less academic knowledge than peers from more affluent families; substantial differences are present even before children start kindergarten. Although these differences in preschoolers' knowledge are present in many subjects, they appear to be especially substantial in knowledge of mathematics (Case, Griffin, & Kelly, 1999).

Crucial to understanding this phenomenon is specifying the types of mathematical knowledge on which the discrepancy is present. On nonverbal numerical tasks, preschoolers' performance does not vary with economic background (Ginsburg & Russell, 1981; Jordan, Huttenlocher, & Levine, 1992; Jordan, Levine, & Huttenlocher, 1994). However, on tasks with verbally stated or written numerals, the knowledge of preschoolers and kindergartners from low-income families lags far behind that of peers from more affluent families. The differences are seen on a wide range of foundational tasks: recognizing written numerals, reciting the counting string, counting sets of objects, counting up or down from a given number other than 1, adding and subtracting, and comparing numerical magnitudes (Ginsburg & Russell, 1981; Griffin, Case, & Siegler, 1994; Jordan et al., 1992; Jordan, Kaplan, Olah, & Locuniak, 2006; Jordan et al., 1994; Saxe, Guberman, & Gearhart, 1987; Starkey, Klein, & Wakeley, 2004; Stipek & Ryan, 1997).

These early differences in mathematical knowledge have an enduring impact. Kindergartners' performance on tests of mathematical knowledge is predictive of mathematical achievement in third, fifth, and eighth grade and even in high school (Duncan et al., 2007; Stevenson & Newman, 1986). This stability over time of individual differences in mathematical knowledge exemplifies the typical positive relation between early and later knowledge (Bransford, Brown, & Cocking, 1999), but the stability of individual differences in math is unusually great. For example, in six longitudinal studies reviewed by Duncan et al. (2007), the standardized beta coefficients relating early and later mathematical knowledge were more than twice as large as the coefficients relating early and later reading proficiency, control of attention, and socioemotional competence. These findings and related ones motivated us to analyze the sources of individual differences in young children's numerical knowledge and to use the analyses to generate methods for helping low-income preschoolers increase their knowledge.

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A Theoretical Analysis of Numerical Magnitude Development

Increases with age and experience in reliance on linear representations of numerical magnitudes seem to play a central role in the development of mathematical knowledge. Both changes in representations of numerical magnitudes and the relation of these changes to the growth of mathematical knowledge have been illustrated in research on number line estimation. The number line estimation task that has been used in this research involves presenting lines with a number at each end (e.g., 0 and 100) and no other numbers or marks in between; the goal is to estimate the location on the number line of a third number (e. g., "Where would 74 go?"). This task is particularly revealing because it transparently reflects the ratio characteristics of the number system. Just as 60 is twice as large as 30, the distance of the estimated position of 60 from 0 should be twice as great as the distance of the estimated position of 30 from 0. More generally, estimated magnitude (y) should increase linearly with actual magnitude (x) with a slope of 1.00, as in the equation $y = x$.

Early in development, however, children's estimates often do not increase linearly with numerical magnitude. Many preschoolers, including ones who can count perfectly from 1 to 10, do not even understand the rank order of those numbers' magnitudes. This poor understanding of the rank order of numerical magnitudes is evident in preschoolers' inaccurate numerical magnitude comparison (Ramani & Siegler, 2008; Whyte & Bull, 2008), number line estimation (Siegler & Ramani, 2008; Whyte & Bull, 2008), and performance on several other measures of numerical magnitude understanding (Condry & Spelke, 2008; Le Corre & Carey, 2007).

Even after children learn the rank order of numbers' magnitudes, they still do not immediately represent the magnitudes as increasing linearly. For example, on numerical magnitude comparison, a task that requires only knowledge of the rank order of numbers' magnitudes, kindergartners from middle-income families are about 90% accurate for numbers between 0 and 100 (Laski & Siegler, 2007). However, the same children's number line estimates in the same numerical range often do not fit any linear function well (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Siegler & Booth, 2004). It is not until second grade that most children generate linearly increasing estimation patterns on 0–100 number lines, and it is not until fourth grade that they do so on 0–1,000 number lines (Booth & Siegler, 2006). The fact that second graders often generate linear estimation patterns for the 0–100 range but nonlinear ones for the 0–1,000 range, despite the tasks being identical except for the numbers being estimated, suggests that experience with the particular range of numbers is crucial to the acquisition of linear estimation patterns. Evidence from the number line estimations of the Mundurcu, an Amazonian indigenous people who usually have little, if any, formal education and whose language has few number words, provides converging evidence that numerical experience, rather than maturation, is crucial to the acquisition of linear representations (Dehaene, Izard, Spelke, & Pica, 2008). Mundurcu adults generate systematic number line estimation patterns, but the estimates increase nonlinearly with increasing numerical magnitude.

The development of linear patterns of number line estimates is not an isolated phenomenon. Rather, it seems to reflect the development of a quite general linear representation of numerical magnitudes. This representation is not invariably used on numerical tasks even in adulthood (Banks & Coleman, 1981; Holyoak & Mah, 1982), but it is used on an increasing range of tasks. Reliance on this linear representation grows to a similar extent between second and fourth grades on three different estimation tasks: number line, numerosity, and measurement estimation (Booth & Siegler, 2006). Degree of linearity of number line estimates correlates positively with overall math achievement test performance at all grade levels between kindergarten and fourth grade, and the same is true with linearity of numerosity and measurement estimation at the two grade levels for which relevant data exist (second and fourth grade; Booth & Siegler, 2006; Geary et al., 2007; Siegler & Booth, 2004). These relations between linearity of estimates on the three estimation tasks and math achievement test scores are substantial, typically ranging between $r = .50$ and $r = .60$ within each grade.

These findings raise the following question: What types of experiences lead children to first represent the magnitudes of verbally stated or written numerals as increasing linearly? Counting experience during the preschool period probably contributes, but such experience appears insufficient to create the linear representations. Children often count perfectly in a numerical range at least a year before they generate linear representations of numerical magnitudes in that range (Le Corre, Van de Walle, Brannon, & Carey, 2006; Ramani & Siegler, 2008; Schaeffer, Eggleston, & Scott, 1974).

If counting experience is insufficient to yield linearly increasing magnitude representations, then what other numerical experiences might contribute? One common activity that seems ideally designed for producing such representations is playing linear numerical board games—that is, board games with linearly arranged, consecutively numbered, and equal-sized spaces (e.g., *Chutes and Ladders*.) Such board games provide multiple cues to both the order of numbers and the numbers' magnitudes. The greater the number in a square, the greater (a) the number of discrete movements of the token that the child has made, (b) the number of number names that the child has spoken, (c) the number of number names that the child has heard, (d) the distance that the child has moved the token, (e) the endpoint of the token's travel, and (f) the amount of time that has passed since the game began. The linear relations between numerical magnitudes and these kinesthetic, auditory, visuospatial, and temporal cues provide a broadly based, multimodal, embodied foundation for a linear representation of numerical magnitudes.

Seen from another perspective, such board games provide a physical realization of the mental number line, hypothesized to be the central conceptual structure for understanding numerical operations in general and numerical magnitudes in particular (e.g., Case & Okamoto, 1996; Dehaene, 1997). Linear number board games also provide children with practice at counting and at numeral identification, at least when players are required to name the squares through which they move (e.g., saying "6, 7" after starting on the 5 and spinning a 2). Thus, playing such games would be expected to improve counting and numeral identification skills as well as performance on tasks that require understanding of numerical magnitudes.

Recent studies have demonstrated the usefulness of this analysis for improving the numerical knowledge of preschoolers from low-income backgrounds (Ramani & Siegler, 2008; Siegler & Ramani, 2008). Playing a linear numerical board game with squares numbered from 1 on the left end to 10 on the right for four 15–20-min sessions yielded the predicted improvements in preschoolers' proficiency on four numerical tasks: numerical magnitude comparison, number line estimation, counting, and numeral identification. The gains remained evident 9 weeks after the posttest (Ramani & Siegler, 2008). Peers who played an identical game, except for the squares varying in color rather than number, did not improve on any of the tasks. Moreover, amount of board game experience outside the laboratory context correlated positively with proficiency on all four tasks, and children from middle-income backgrounds reported playing board games (but not video games) at home and at the homes of friends and relatives more often than preschoolers from low-income families. Together, these findings strongly suggest that differences among individuals and socioeconomic groups in experience playing board games contribute to differences in early numerical knowledge. In the present study, we tested not only whether the linearity of the board game is important for building a linear representation but also whether representing numerical magnitudes linearly enhances learning of novel arithmetic problems.

The Present Study

We pursued four main goals in this study. The first was to test the representational mapping hypothesis: The greater the transparency of the mapping between physical materials and desired internal representations, the greater the learning of the desired internal representation. As numerous students of analogical reasoning have noted (e.g., Doumas, Hummel, & Sandhofer, 2008; Gentner & Markman, 1997; Goswami, 2001; Holyoak & Thagard, 1995; Richland, Morrison, & Holyoak, 2006), physical representations that capture key structural features of desired internal representations and map onto them in transparent ways are often particularly compelling. They are both easier to acquire and easier to remember than analogies in which the mapping between physical and internal representations is less direct.

The implication of the representational mapping hypothesis in the present context is that if the desired internal representation of numerical magnitudes is a linear number line, then playing the number game with a linear board should promote greater learning of numerical magnitudes than playing the identical game with a circular board. This prediction rests on the assumption that children find it easier to form linear than circular representations of numerical magnitudes. If this assumption is false, then it should be as easy to form a circular representation of numerical magnitude in response to experience with a circular board as to form a linear representation in response to experience with a linear board. However, there were several reasons to hypothesize that the linear representation is easier to acquire than a circular representation.

A great deal of evidence suggests that adults and older children usually represent numerical magnitudes in a form akin to a mental number line. Dehaene, Bossini, and Giraux (1993, p. 394) stated this view succinctly:

> A representation of number magnitude is automatically accessed during parity judgments of Arabic digits. This representation may be likened to a mental number line (Restle, 1970), because it bears a natural and seemingly irresistible correspondence with the left–right coordinates of external space.

If, as Dehaene et al. suggested, something akin to a horizontally oriented mental number line is the natural way of representing numerical magnitudes, then the representational mapping hypothesis implies that playing the game on a linear board should promote greater learning of numerical magnitudes than playing the game on a circular board.

However, another hypothesis is also plausible. It may be as easy to form a circular representation as a linear one if relevant experience is provided. Circular external representations sometimes are used to depict numerical magnitudes (e.g., analog clocks, speedometers, scales for weighing food). In addition, adults can generate and use circular internal representations of numerical magnitudes if asked to do so (Bächtold, Baumüller, & Brugger, 1998), and children also might be able to do so if asked. From this perspective, the common use of linear representations described by Dehaene et al. (1993) and many others may reflect the experiences through which children typically learn about numerical representations rather than the notion that linear representations are easier to learn.

The present examination of low-income preschoolers' reactions to linear and circular board games provided an atypical but valuable type of evidence about whether linear representations of numerical magnitudes are indeed more natural than circular ones—evidence regarding learning. If one representation is easier to form than another, then children should more efficiently learn that representation from relevant experiences. In the present context, if the internal linear representation is easier to form than the internal circular representation, then children who initially do not use either representation should learn more from experience with linear external representations of numerical magnitudes than from experience with circular external representations of the same numerical magnitudes.

The comparison between learning from linear and circular boards is particularly interesting because the circular board includes several of the same cues to magnitudes that are available on the linear board—auditory, kinesthetic, and temporal cues—and the validities of these cues are identical on the two boards. The only cues that that are different when playing the game on the two boards are those of linear distance traveled and spatial end point in the direction of motion. However, such spatial cues are highly salient for young children, as indicated by their strong reliance on them on number conservation, liquid and solid quantity conservation, relative time judgment, and relative speed judgment tasks (Levin, 1977; Piaget, 1952; Siegler & Richards, 1979). This tendency to rely on spatial cues to judge relative numbers, amounts, times, and speeds may be part of a general tendency to think of quantitative dimensions in ways akin to a mental number line oriented horizontally in space.

Extending the representational mapping hypothesis further, the greater learning that is predicted for children who play the game with the linear board should be specific to tasks that assess knowledge of numerical magnitudes. There was no reason to predict that the linear board would promote greater improvement than the circular board in counting or in numeral identification because those skills do not depend in any obvious way on a linear representation of numerical magnitudes. Instead, the linear board and the circular board were expected to be equally effective in promoting learning of counting and numeral identification because playing the game on them requires identical counting and numeral identification activities. No cues that seemed likely to contribute to either skill were present on one board but not the other. Thus, the first major prediction of this study was that experience with a linear board would produce greater learning than experience with a circular board on number line estimation and numerical magnitude comparison tasks.

The second major goal of the present study was to test the prediction that forming a linear representation of numerical magnitudes should improve young children's ability to learn answers to arithmetic problems. As noted previously, the knowledge of numerical magnitudes of many preschoolers from low-income backgrounds is very poor. Even their knowledge of the rank order of numerical magnitudes is shaky. When comparing the magnitudes of pairs of numbers from 1–9, a task on which perfect accuracy requires only knowledge of the rank order of the numbers, the low-income 4- and 5-year-olds in Ramani and Siegler's (2008) study only answered 70% of problems correctly (vs. a 50% chance level). If children do not even know the rank order of the magnitudes of numbers, then learning arithmetic is reduced to learning nonsense syllables; there is no more reason that $3 + 3 = 6$ than $3 + 3 = 2$.

Linear representations of numerical magnitudes seem likely to help children learn arithmetic because such representations maintain equal subjective spacing throughout the entire range of numbers, thus facilitating discrimination among answers to different problems. At least two prior findings are consistent with the hypothesis that linear numerical magnitude representations facilitate arithmetic learning. First, linearity of number line estimates is positively correlated with arithmetic proficiency among first through fourth graders (Booth & Siegler, 2006, 2008). Second, representing the magnitudes involved in arithmetic problems by displaying on a computer screen horizontally oriented bars whose lengths are proportional to the sizes of addends and sums facilitates first graders' learning of the sums (Booth & Siegler, 2008). Playing the linear number board game was expected to produce a greater increase in correct answers and also errors that are closer to the correct answer than playing the circular board game.

Increasing percentages of "close miss" errors among children who had played the linear board game, both in absolute terms and relative to children in the other two groups, would be particularly strong evidence of the importance of numerical magnitude representations in learning arithmetic. Why other than an improved sense of numerical magnitudes would children who earlier had played the linear board game increasingly retrieve answers to arithmetic problems that were wrong but close to the right answer? Why else would the same trends not be seen among children in the other two groups who received the identical arithmetic training procedure?

A third goal of the present study was to examine whether playing the board games produced greater learning than engaging in other types of numerical activities. Prior studies used a control condition in which children played the same board game but with colors rather than numbers in the squares. Thus, a child might spin a spinner and say, "red, blue," rather than "7, 8." This condition controlled for a variety of plausible alternative hypotheses—time spent interacting with the experimenter, time spent moving a token in a careful one-square-at-a-time manner, intervening numerical experience in the classroom—but it did not indicate whether other numerical experiences of the types that occur most often in preschools might be just as useful or more useful than playing the linear number board game.

Three relatively frequent numerical activities in preschool are reciting the counting string, counting objects, and naming numerals (Ginsburg & Russell, 1981; Saxe et al., 1987). These are often conducted as group activities, and feedback regarding an individual child's correctness or incorrectness does not appear to be common. To determine whether playing numerical board games had effects above and beyond those of such common preschool activities, we provided children in the numerical control activities condition with counting and numeral naming tasks similar to those that are common in preschool classrooms. The prediction was that playing linear numerical board games would lead to greater learning of numerical magnitudes than engaging in these control activities, which do not require understanding of numerical magnitudes. Thus, playing the linear numerical board game was expected to lead to superior performance on the number line, magnitude comparison, and arithmetic learning tasks. No prediction was made for the numeral identification and counting tasks because all conditions required children to engage in such activities.

A fourth goal of this study was to more deeply understand how individual differences influence learning of numerical information. This goal subsumed two related issues: stability of individual differences over the course of learning for the entire sample and relations of the learning of children below and above the median in initial knowledge. Relevant to the first issue, Ramani and Siegler (2008) found that individual differences in pretest performance were stable on both a posttest that followed 2 weeks of linear board game experiences and on a follow-up test 2 months later. That is, the same children who scored highest on the pretest also scored highest after the learning experiences, even though both children toward the top of the distribution and toward the bottom learned a considerable amount between pretest and posttest. We wanted to test whether the same stability of individual differences would be present under the two novel experimental conditions—circular number board game and numerical control activities—that were examined in the present study.

The second issue was the relative learning of children above and below the median in initial knowledge. Even if the same children were highest in numerical knowledge before and after game playing experience, the gap between children of greater and lesser knowledge might increase, decrease, or remain unchanged. Learning might be greatest among children who already have a reasonable amount of numerical knowledge (the often observed rich get richer effect), it might be greatest among children whose initial knowledge is especially poor (a catching up effect, perhaps because of the experimental experience constituting a higher proportion of these children's total numerical experience), or it might be independent of initial knowledge (similar experience leading to similar learning). This issue was important for practical as well as theoretical reasons; the findings could help determine whether an identifiable subset of children, in particular those with little initial knowledge, did not learn much from playing the board games. Such a finding would trigger efforts to improve the games so that these children, too, would benefit.

Method

Participants

Participants were 88 preschoolers (56% female) ranging in age from 4 years, 0 months to 5 years, 5 months (M = 4 years, 8 months, SD = 0.47). Among them, 34% were African American, 61% were Caucasian, and 5% were Asian, Hispanic, biracial, or unknown. The children were recruited from seven Head Start classrooms and two childcare centers, all of which served families with very low incomes. The families from the Head Start classrooms met the income requirements for Head Start established by the federal government for 2007 (e.g., for a family of three, annual income below $17,170). Almost all of the other families (96%) received government subsidies for childcare expenses.

Children within each Head Start or childcare center were randomly assigned to one of three conditions: the linear board game condition, the circular board game condition, or the numerical control condition. The linear board condition included 30 children (M = 4 years, 8 months, SD = 0.46; 60% female; 40% African American, 53% Caucasian, 7% Asian, Hispanic, biracial, or unknown). The circular board game condition included 29 children (M = 4 years, 8 months, SD = 0.45; 52% female; 31% African American, 62% Caucasian, 7% Asian, Hispanic, biracial, or unknown). The numerical activities control condition included 29 children (M = 4 years, 8 months, SD = 0.52; 55% female; 31% African American, 69% Caucasian). An additional 3 children (2 in the circular board game condition and 1 in the numerical activities control condition) were present for the pretest but did not complete the experiment because they were absent for an extended period. The experimenters were a female postdoctoral research associate of Indian descent (Geetha B. Ramani) and a female Caucasian research assistant.

Materials and Procedure

All of the preschoolers met individually with an experimenter for five 15–20 min sessions within a 3-week period. Sessions were held in either their classroom or an unoccupied room nearby. Each experimenter met with the same children for all sessions in the study and with approximately equal numbers of children in each of the three conditions.

Linear board game condition. A board 52 cm wide and 24 cm high was used in the linear board game condition. The name of the game, *The Great Race*, was printed at the top of the board. Below the name were 10 equal-sized squares of different colors arranged in a horizontal array. Each square contained one number, with the numerical magnitudes increasing from left to right. The word "Start" was just to the left of the "1" square; the word "End" was just to the right of the "10" square.

The game also included a spinner with a "1" half and a "2" half, as well as a bear token and a rabbit token. The child chose the bear or rabbit token before each session to represent his or her progress on the board; the experimenter took the remaining token. Because of children almost always choosing to go first, and that being a substantial advantage in this game, children won most games.

At the beginning of each session, the experimenter told the child that they would take turns spinning the spinner and that whoever reached the end first would win. Then the experimenter said that on each turn, the player who spun the spinner would move her or his token the number of spaces indicated on the spinner. The experimenter also told the child to say the numbers on the spaces through which the token moved. Thus, children who were on the square with a 3 and spun a 2 would say, "4, 5," as they moved.

If a child erred or could not name the numbers, then the experimenter correctly named them and the child then had to repeat the numbers while moving the token. One common error involved children not naming the numbers in the squares as they moved their token and instead counting the number of squares they moved their token forward. Children who made this error would, if they were on the fourth square and spun a 2, say "1, 2" as they moved their token instead of "5, 6." When children erred in this way, the experimenter reminded them to name the numbers in the squares as they moved. If the child did not correct the error, then the experimenter would point to and name the numbers in the squares and have the child repeat them as she (the experimenter) pointed to the squares. Preschoolers played the game approximately 20 times, with each game lasting about 3 min. Children were not told explicitly "that's right" or "that's wrong," but the correction procedures in both the linear and circular board game conditions provided implicit feedback.

Circular board game condition. The only difference between the linear and circular board game conditions was the board itself. There were two circular boards, each divided into 12 wedges. Both were 38 inches (96.52 cm) high and 41 inches (104.14 cm) wide. Ten of the wedges, those located approximately at the locations of 2:00 through 10:00 on an analog clock, included the numbers 1–10 ordered consecutively. On one board, the numbers increased clockwise; on the other, the numbers increased counterclockwise. Also on the circular board were two wedges of the same size at the top of the board that did not contain numbers and that separated the numbers 10 and 1. One of these wedges contained a picture of a tree and the word "Start;" the other contained a picture of a trophy and the word "Finish." The color of the wedge in which each number appeared was the same as on the corresponding square of the linear board. The procedure followed in the circular board condition was identical to that in the linear board condition. Half of the children in this condition played the clockwise version of the game (n = 15), and the other half played the counterclockwise version (n = 14). As with the linear board game condition, preschoolers played the circular board game approximately 20 times over Sessions 1–4, with each game lasting about 3 min.

Numerical activities control condition. Preschoolers in the numerical activities control condition were presented three tasks in a continuing cycle: number string counting, numeral identification, and object counting. Whichever activity would have been next at the end of one session was first at the following session. On the object counting task, children were asked to count a row of between 1 and 10 poker chips, with the exact number varying randomly. The procedures for the other two tasks—number string counting and numeral identification—were the same as those used to assess those skills on the pretest and posttest; they are described in the next section.

Each child in the numerical activities control condition was matched with a child of the same age (within 2 months) in one of the board game conditions. The length of each session for each child in the numerical control condition was equated with that of the matched child. Thus, if a child in a board game condition played the game for 16 min in Session 2, the matched child in the numerical activities control condition also engaged in those activities for 16 min in Session 2. General praise and encouragement were presented periodically, but no specific feedback regarding correctness was presented in this condition.

Measures of Numerical Knowledge

At the beginning of Session 1, children were administered a pretest that included five numerical tasks presented in the following order: counting, number line estimation, magnitude comparison, numeral identification, and arithmetic. At the end of Session 4, a posttest was presented that included the first four of these tasks in the same order as on the pretest. The posttest for the remaining task (arithmetic) was presented at the end of Session 5, after children had been presented opportunities to learn previously unknown answers to arithmetic problems.

Counting. Children were asked to count from 1 to 10. Counting was coded as correct up to the first error (e.g., if a girl counted "1, 2, 3, 4, 5, 6, 8, 9, 10," her score was 6).

Number line estimation. Children were presented 18 sheets of paper, one at a time. On each sheet was a 25-cm line, with "0" just below the left end and "10" just below the right end. A number from 1–9 inclusive was printed approximately 2 cm above the center of the line, with each number printed on 2 of the 18 sheets. All numbers from 1–9 were presented once before any number was presented twice; the nine numbers were ordered randomly both times. Children were told that they would be playing a game in which they needed to mark the location of a number on a line. On each trial, after asking the child to identify the number at the top of the page (and helping if needed), the experimenter asked, "If this is where 0 goes (pointing) and this is where 10 goes (pointing), where does *N* go?"

Numerical magnitude comparison. Children were presented a 20-page booklet, each page displaying two numbers between 1 and 9 inclusive, and asked to choose the bigger number. The experimenter first presented two warm-up problems with feedback, followed by 18 experimental problems without feedback. The 18 experimental problems were a randomly chosen half of the 36 possible pairs. On the warm-up problems, the experimenter pointed to each number and asked, for example, "John (Jane) had one cookie and Andy (Sarah) had six cookies. Which is more: one cookie or six cookies?" On the two warm-up problems, the experimenter corrected any errors that were made (e.g., "Actually, six cookies is more than one") and repeated the problems until the child answered them correctly. On the 18 experimental problems, half of the children within each condition were presented a given pair in one order (e.g., "Is six cookies more than three cookies?") and half in the opposite order ("Is three cookies more than six cookies?").

Numeral identification. The task involved 10 randomly ordered cards, each with a numeral from 1–10 on it. On each trial, the experimenter held up a card and asked the child to name the numeral.

Arithmetic problems and training. The arithmetic pretest was composed of four addition problems presented in the following order: 2 + 1, 2 + 2, 4 + 2, and 2 + 3. Children were asked, "Suppose you have *N* oranges and I give you *M* more; how many oranges would you have then?" As on the other pretest and posttest tasks, no feedback was given.

At the beginning of Session 5, children received training on the first two arithmetic problems that they had answered incorrectly on the pretest. The training involved presenting the two problems and their answers three times in alternating order. For example, children who erred on all four problems on the pretest were presented 2 + 1 and 2 + 2 in the first cycle of Session 5, 2 + 2 and 2 + 1 in the second cycle, and 2 + 1 and 2 + 2 in the third cycle. The problems were presented in the same "oranges" context as on the pretest. Children needed to answer each problem within 5 s; if they failed to do this, they were prompted to answer. On each trial, after children stated their answer, they were asked to explain how they obtained that answer. Then, they were given feedback and told the right answer. For example, on 2 + 2, they were told either "That's right; 2 + 2 is 4," or "No, 2 + 2 is 4." The children's explanations indicated that on almost all trials (96%), they retrieved the answer from memory or guessed. After the third cycle of feedback problems, children received the addition posttest, in which they were presented the same four problems in the same order as on the pretest and asked to state the answer.

Results

Preliminary analyses comparing the performance of children who used the clockwise and the counterclockwise circular boards indicated no differences on any measure. Children who played the clockwise version scored directionally higher on three posttest tasks and directionally lower on the other two. Therefore, no distinction between the two circular boards was made in further analyses. Preliminary analyses examining age differences also did not reveal any differences; therefore, age was not included in further analyses.

Multivariate Analyses

We first examined multivariate effects of condition and session across number line estimation, magnitude comparison, counting, and numeral identification tasks (arithmetic performance was not included because the multivariate analysis of variance was intended to measure direct effects of playing the board games, and arithmetic performance reflected subsequent experience with the addition problems). For the magnitude comparison and numeral identification tasks, the dependent measure was number of correct answers. For the counting task, the dependent measure was number of numbers counted correctly before the first error. For the number line estimation task, two measures—linearity and slope—were included because they provide somewhat different types of information. The ideal function relating actual and estimated magnitudes on the number line test is perfectly linear ($R^2_{\text{lin}} = 1.00$) with a slope of 1.00. However, estimates can increase in a perfectly linear function with a slope far less than 1.00, and estimates can increase with a slope of 1.00 but not fit a linear function very closely. For this reason, both measures were included.

A 3 (condition: linear board game, circular board game, or numerical activities control) × 2 (session: pretest or posttest) repeated-measures multivariate analysis of variance was conducted on the five measures described above. Effects emerged for session, $F(5, 81) = 13.26$, $p < .001$, $\eta^2_p = .45$, and the Condition × Session interaction, $F(10, 164) = 3.83$, $p < .001$, $\eta^2_p = .19$. To better understand the interaction, and to examine the consistency of results across tasks, we conducted univariate analyses for each task.

Number Line Estimation

Linearity. Linearity of number line estimation was the measure that most directly corresponded to the hypothetical construct of a linear representation of numerical magnitude. Among children who played the linear board game, the variance in the group median estimates for each number that was accounted for by the best fitting linear equation increased substantially from pretest (22%) to posttest (94%). As shown in Figures 1, 2, and 3, changes were much smaller among children who played the circular board game (11% vs. 26%) or who participated in the numerical activities control condition (43% vs. 57%).

Analyses of individual performance provided converging evidence. Linearity of individual children's number line estimates varied with session, $F(1, 85) = 21.92$, $p < .001$, $\eta_p^2 = .21$, and with the Condition × Session interaction, $F(2, 85) = 8.52$, $p < .001$, $\eta_p^2 = .17$. The interaction between condition and session resulted from the linear board game producing greater pretest–posttest improvement than the other two conditions. Among children who played the linear board game, the mean percentage of variance in individual children's estimates that was accounted for by the best fitting linear function increased from 14% on the pretest to 39% on the posttest, $t(29) = 4.75$, $p < .001$, $d = 1.03$. In contrast, there were no significant changes among children who played the circular board games (15% vs. 21%) or who participated in the numerical activities control condition (16% vs. 18%). Viewed from another perspective, the linearity of pretest estimates of children in the three conditions did not differ (mean $R^2_{\text{lin}} = .14$, .15, and .16), but the posttest estimates of children who had played the linear board game were considerably more linear than those of children who had played the circular board game (mean $R^2_{\text{lin}} = .39$ vs. .21), $t(57) = 2.50$, $p < .05$, $d = 0.65$, or who had engaged in the numerical control activities (mean $R^2_{\text{lin}} = .39$ vs. .18), $t(57) = 2.90$, $p < .01$, $d = 0.76$. Linearity of estimates did not differ for the latter two conditions.

Slope. Among children who played the linear board game, the slope of the group median estimates for each number increased substantially from pretest to posttest (.03 vs. .78). No comparable changes in slope occurred among children who played the circular board games (.03 vs. .12) or among children who participated in the numerical control activities (.05 vs. .09; see Figures 1, 2, and 3).

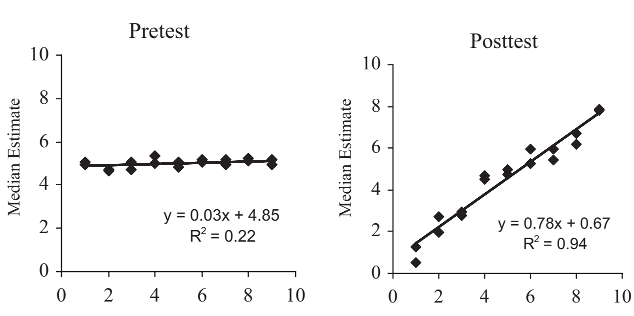

Figure 1. Pretest and posttest linearity of number line estimation in the linear board game condition: group median data.

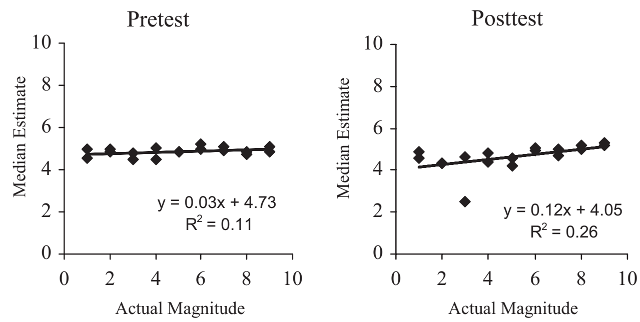

Figure 2. Pretest and posttest linearity of number line estimation in the circular board game condition: group median data.

Analyses of the slopes of individual children's number line estimates provided converging evidence for the group-level pattern. As in the analyses of linearity, the slopes varied with session, $F(1, 85) = 38.16$, $p < .001$, $\eta_p^2 = .31$, and with the Condition × Session interaction, $F(2, 85) = 11.16$, $p < .001$, $\eta_p^2 = .21$. The interaction between condition and session once more reflected larger gains among children who played the linear board game. In this group, the mean slope of number line estimates increased substantially from pretest to posttest (mean slope = .04 vs. .61), $t(29) = 5.65$, $p < .001$, $d = 1.26$. Among children who played the circular board games, the slopes of number line estimates increased from pretest to posttest, though less dramatically (mean slope = .09 vs. .28), $t(28) = 2.34$, $p < .05$, $d = 0.48$. Among children in the numerical control condition, pretest–posttest changes were not significant (.12 and .20). Viewed from another perspective, slopes in the three conditions did not differ on the pretest (mean slope = .04, .09, and .12). However, on the posttest, the slopes of the estimates of children who had played the linear board game were higher than those of children who had played the circular board game (mean slope = .61 vs. .28), $t(57) = 2.71$, $p < .01$, $d = 0.71$, or who had participated in the control condition (mean slope = .61 vs. .21), $t(57) = 3.37$, $p < .01$, $d = 0.88$. Again, slopes did not differ between children who had played the circular board game and children who had engaged in the numerical control activities.

Accuracy. To obtain a composite measure of the accuracy of children's number line estimates, and to allow comparison with previous studies, we examined percent absolute error (PAE): PAE = [(Estimate – Actual Number)/Scale of Estimates] × 100. For example, if a child marked the location of 5 on a 0–10 number line at the position that corresponded to 9, the PAE would be 40%: [(9 − 5)/10] × 100.

Accuracy of number line estimation varied with session, $F(1, 85) = 25.71$, $p < .001$, $\eta_p^2 = .23$, and with the Condition × Session interaction, $F(2, 85) = 3.77$, $p < .05$, $\eta_p^2 = .08$. Among preschoolers who played the linear board game, PAE decreased substantially from pretest to posttest (29% to 21%), $t(29) = 4.85$, $p < .001$, $d = 1.01$. Among preschoolers who played the circular board game, a smaller but significant improvement also was present (29% to 26%), $t(28) = 2.14$, $p < .05$, $d = 0.43$. PAE of preschoolers in the numerical control condition did not change significantly from pretest to posttest (28% and 25%). Pretest PAE of children in the three conditions was comparable (29%, 29%, and

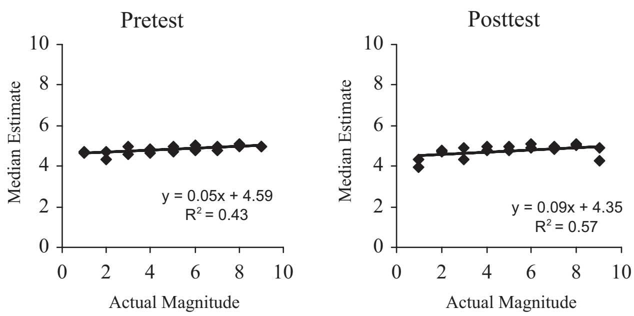

Figure 3. Pretest and posttest linearity of number line estimation in the numerical control activities condition: group median data.

28%), but posttest PAE of children who had played the linear board game was superior to posttest PAE of both those who had played the circular board game (21% vs. 26%), $t(57) = 2.59$, $p < .05$, $d = 0.67$, and those who had engaged in the numerical control activities (21% vs. 25%), $t(57) = 2.41$, $p < .05$, $d = 0.63$. Posttest PAE did not differ for the circular board game and numerical control activities conditions.

Numerical Magnitude Comparison

Number of correct magnitude comparisons varied with the Condition × Session interaction, $F(2, 85) = 5.43$, $p < .01$, $\eta_p^2 = .12$. This interaction resulted from the linear board game producing greater gains in magnitude comparison accuracy than the circular board game or the numerical control activities. Among children who played the linear board game, percent correct magnitude comparisons increased from 68% on the pretest to 77% on the posttest, $t(29) = 3.79$, $p < .01$, $d = 0.51$. In contrast, there was no change over sessions in magnitude comparison accuracy for children who played the circular board game (70% correct on the pretest and 71% correct on the posttest) or who participated in the numerical control condition (69% correct on the pretest and 65% correct on the posttest). Magnitude comparison performance on the pretest did not differ among the three conditions (68%, 70%, and 69% correct), but on the posttest, children who had played the linear board game answered more magnitude comparison items correctly than did children in the numerical control activities condition (77% vs. 65%), $t(57) = 2.88$, $p < .01$, $d = 0.75$. There was no difference on the posttest in magnitude comparison accuracy for the circular board game and numerical control conditions.

Counting

There were no significant effects on the counting task. The reason was simple—almost all children in all three conditions were at ceiling on both the pretest and the posttest. That is, almost all children counted to 10 without an error in the linear board game condition (90% of children on the pretest and 93% on the posttest), the circular board game condition (90% of children on the pretest and 97% on the posttest), and the numerical control condition (86% of children on both pretest and posttest).

Numeral Identification

Number of correct numeral identifications varied with session, $F(1, 85) = 18.80$, $p < .001$, $\eta_p^2 = .18$, and with the Condition × Session interaction, $F(2, 85) = 3.21$, $p = .05$, $\eta_p^2 = .07$. The interaction between condition and session resulted from differing gains in numeral identification among children in the three conditions. Children who played the linear board game improved from a mean of 6.0 correct identifications of numerals on the pretest to 7.3 correct identifications on the posttest, $t(29) = 3.95$, $p < .001$, $d = 0.47$. Children who played the circular board games improved from 6.6 to 7.3 correct identifications from pretest to posttest, $t(28) = 3.17$, $p < .01$, $d = 0.23$. In contrast, the numeral identification skills of children in the numerical control condition did not improve: 6.6 correct on the pretest and 6.8 correct on the posttest. There was no difference among the three conditions on the pretest (6 vs. 6.6 vs. 6.6 correct identifications) or on the posttest (7.3 vs. 7.3 vs. 6.8). However, the pretest–posttest changes suggested that playing both linear and circular board games led to improvements in numeral identification.

Arithmetic

In analyzing the arithmetic data, we examined both number of correct answers and absolute error (the absolute value of the distance of the child's response from the sum). These analyses were limited to the two items on which children received training; there were no differences in performance on the two nontrained items.

To determine whether prior experience influenced subsequent learning of answers to arithmetic problems, we first examined whether the number of arithmetic problems that children in the three groups answered correctly differed after the arithmetic training. Two children were excluded from these analyses—in one case because the child answered all four problems correctly on the pretest and in the other because of experimenter error.

Children who earlier had played the linear board game answered more addition problems correctly after training on them than did children who earlier had played the circular board game (45% vs. 30% correct) or children who earlier had engaged in the numerical control activities (45% vs. 28% correct), $\chi^2(4, N = 86) = 11.46$, $p < .05$. Number of correct answers in the circular board game and numerical activities control groups did not differ. The absolute amount of learning by children in the linear board game group—from 0% correct on the pretest to 45% correct on the posttest—was quite impressive given the brevity of the training.

Analyses of absolute error on the arithmetic problems yielded similar results. Among children who had played the linear board game, the absolute error on the arithmetic problems decreased from pretest to posttest (mean error = 2.3 vs. 1.1), $t(29) = 2.93$, $p < .01$, $d = 0.78$. In contrast, there was no change in absolute error among children who had played the circular board game (mean error = 1.9 vs. 1.7) or among those who had engaged in the numerical control activities (mean error = 2.2 vs. 3.6). No differences in mean absolute error were present among the three groups on either the pretest or the posttest (pretest *Ms* = 2.3, 1.9, and 2.2; posttest *Ms* = 1.1, 1.7, and 3.6). The large difference in posttest means made the lack of a significant difference surprising. The main reason appeared to be that 2 children in the numerical activities control group generated extremely inaccurate estimates on the posttest, which inflated the within-group variance. When a square-root transformation was applied to all of the data to reduce variability, posttest addition answers of children who had played the linear board game tended to be more accurate than those of peers who had engaged in the numerical control activities, $t(57) = 1.88$, $p = .07$, $d = 0.49$.

These analyses include trials that children answered correctly (where their absolute error was 0) as well as trials that involved errors. To determine whether there were differences in the quality of errors per se, we compared the absolute error on the 55% least accurate answers in each condition on pretest and posttest (the percentage of answers that were erroneous for all three groups at both times of measurement). A Kruskal–Wallis test indicated that on the pretest, there were no differences in the quality of errors of children in the three conditions, $H(2) = 2.66$, mean ranks = 51.62, 41.98, and 49.91 for the linear, circular, and control conditions, respectively. In contrast, on the posttest, errors of children in the linear board group were closer misses than those in the other conditions, $H(2) = 12.81$, $p < .01$, mean ranks = 34.59, 54.10, and 56.11 for the linear, circular, and control conditions, respectively. Among children in the linear board game condition, absolute error on the 55% least accurate answers decreased from pretest to posttest (3.33 to 2.00), Wilcoxon test, $Z = 5.21$, $p < .001$. In contrast, among children in the other two groups, absolute errors on the 55% least accurate answers did not change from pretest to posttest. Thus, from pretest to posttest, percentage of correct addition answers increased and errors tended to become closer to the correct answer among children who had previously played the linear numerical board game but not among children in the other two conditions.

The results of the arithmetic training suggest that improvements in children's numerical magnitude knowledge from playing the linear board game aided their learning of the addition problems from the arithmetic training. To examine the contribution of numerical magnitude knowledge to learning answers to addition problems, we converted the linearity of individual children's number line estimates and the number of correct magnitude comparisons at posttest to *z* scores and the two *z* scores were summed to create a composite measure of magnitude knowledge. A linear regression was conducted using this composite measure to predict absolute error on the addition problems at posttest. This composite measure of numerical magnitude knowledge predicted the absolute error at posttest ($R^2 = .04$), $F(1, 85) = 3.35$, $p = .04$, one-tailed.

Individual Differences

To examine the stability of individual differences in numerical knowledge from pretest to posttest, and to determine whether the experimental manipulations altered individual differences within each condition, we computed pretest–posttest correlations separately for each condition. The individual differences proved quite stable in all three conditions. As shown in Table 1, this was especially the case in the numerical activities control group. In that condition, children's relative proficiency was highly stable for all four tasks and all five measures. Given that numerical knowledge changed less in this condition than in the others, the greater stability of individual differences is not entirely surprising. Nonetheless, the absolute amount of stability of the individual differences was striking, if for no other reason than demonstrating the reliability of the measures.

Individual differences were also quite stable from pretest to posttest in the two experimental conditions in which greater learning occurred: the linear and circular board game conditions. In both of those conditions, performance on three of the four tasks showed substantial stability (see Table 1). The exception in the circular board game condition was counting performance, an exception that was attributable to ceiling effects (90% of children in that condition counted perfectly on the pretest and 97% on the posttest). The exception in the linear board game condition involved number line performance; the reason why individual differences in that condition on that task were less stable than in the other conditions and on the other tasks may have been due to floor effects on the pretest (half of the children generated number line slopes on the pretest that were negative).

Table 1
Pretest–Posttest Correlations for Each Task in Each Condition

Task	Numerical control group	Circular board game group	Linear board game group
Numeral identification	$.84^{***}$	$.93^{***}$	$.80^{***}$
Magnitude comparison	$.56^{**}$	$.74^{***}$	$.69^{***}$
Counting	$.73^{***}$	.04	$.66^{***}$
Number line linearity	$.88^{***}$	$.60^{**}$	.33
Number line slope	$.83^{***}$	$.38^{*}$	.27

$^{*} p < .05$. $^{**} p < .01$. $^{***} p < .001$.

Existing Knowledge and Acquisition of New Knowledge

Who benefited most from playing the linear board game? To find out, we divided children in the linear board game condition, the condition that produced consistent improvement on all measures, into those performing above and below the median on the pretest. Then we compared pretest and posttest performance on each task of children in each subgroup. The pattern of changes on the four tasks is shown in Figure 4.

Number line linearity. The 30 participants in the linear board game condition were divided into a group of 16 children for whom the best fitting linear function accounted for 8% or less of the variance in pretest number line estimates ($M = 4\%$) and a group of 14 children for whom the best fitting linear function accounted for 9% or more of the variance in pretest estimates ($M = 26\%$). The linearity of individual children's number line estimates varied with session, $F(1, 28) = 23.04$, $p < .001$, $\eta_p^2 = .45$. It increased from pretest to posttest both for those who scored below the median on the pretest (4% to 37%), $t(15) = 4.47$, $p < .001$, $d = 1.59$, and for those who scored above it (26% to 41%), $t(13) = 2.29$, $p < .05$, $d = 0.58$. The two groups differed on the pretest (4% vs. 26%), $t(28) = 4.17$, $p < .001$, $d = 1.55$, but not on the posttest (37% vs. 41%), reflecting larger gains in the group that performed less well on the pretest.

Number line slope. The median split on pretest performance resulted in a group of 15 children with below average slopes ($M = -0.23$, $SD = 0.15$, range $= -0.48$ to -0.02) and a group of 15 children with above average slopes ($M = 0.30$, $SD = 0.31$, range $= -0.01$ to 1.01). Among children who played the linear board game, slope varied with pretest performance, $F(1, 28) = 5.31$, $p < .05$, $\eta_p^2 = .16$; session, $F(1, 28) = 38.83$, $p < .001$, $\eta_p^2 = .58$; and the Pretest Performance × Session interaction, $F(1, 28) = 7.22$, $p < .01$, $\eta_p^2 = .21$. Slopes increased from pretest to

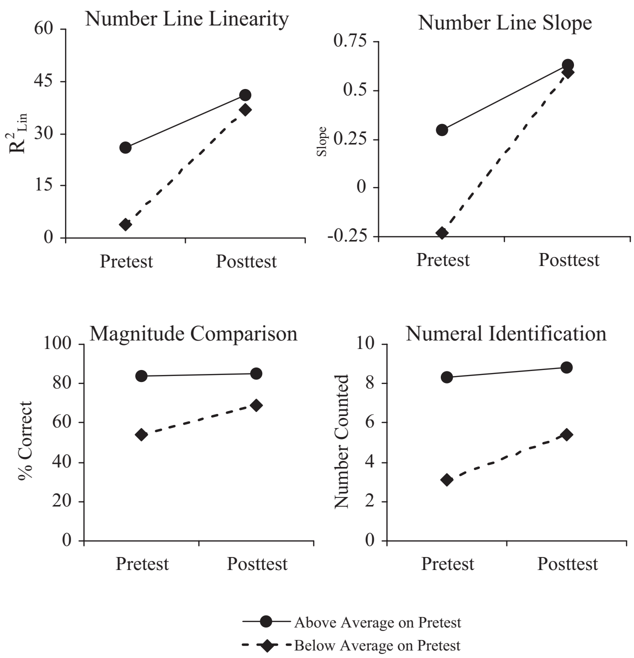

Figure 4. Pretest and posttest performance of children who generated below average or above average pretest performance.

posttest for those whose pretest slope was below the median (mean slope = −0.23 vs. 0.59), $t(14) = 6.12$, $p < .001$, $d = 2.14$, and for those whose pretest slopes were above the median (mean slope = 0.30 vs. 0.63), $t(14) = 2.59$, $p < .05$, $d = 0.72$. The interaction reflected greater improvement from pretest to posttest among children whose initial performance was below the median. Slopes of children in the two groups differed on the pretest (mean slopes = −0.23 vs. 0.30), $t(28) = 5.89$, $p < .001$, $d = 2.15$, but not on the posttest (mean slopes = 0.59 and 0.63).

Numeral identification. Children were divided into a group of 13 with poorer initial numeral identification skills (6 or fewer correct identifications; $M = 3.1$) and a group of 17 with better initial numeral identification skills (7 or more correct identifications; $M = 8.3$). Number of correct numeral identifications varied with pretest performance, $F(1, 28) = 64.13$, $p < .001$, $\eta_p^2 = .70$; session, $F(1, 28) = 23.38$, $p < .001$, $\eta_p^2 = .46$; and the Pretest Performance × Session interaction, $F(1, 28) = 9.19$, $p < .01$, $\eta_p^2 = .25$. Children whose pretest performance was below the median improved from 3.1 to 5.4 correct identifications, $t(12) = 3.90$, $p < .01$, $d = 1.07$, and children whose pretest performance was above the median improved from 8.3 to 8.8 correct identifications, $t(16) = 2.17$, $p < .05$, $d = 0.43$. The two groups differed on both the pretest (3.1 vs. 8.3 correct identifications), $t(28) = 9.19$, $p < .001$, $d = 3.38$, and the posttest (5.4 vs. 8.8 correct identifications), $t(28) = 5.22$, $p < .001$, $d = 1.91$, though the pretest differences were larger.

Numerical magnitude comparison. Children were divided into a group of 16 with 67% or fewer accurate magnitude comparisons on the pretest ($M = 54\%$) and a group of 14 with above 67% correct comparisons ($M = 84\%$). Number of correct comparisons varied with group, $F(1, 28) = 50.59$, $p < .001$, $\eta_p^2 = .64$; session, $F(1, 28) = 17.54$, $p < .001$, $\eta_p^2 = .39$; and the Group × Session interaction, $F(1, 28) = 11.56$, $p < .01$, $\eta_p^2 = .29$. The two groups differed on both the pretest (54% vs. 84% correct), $t(28) = 8.40$, $p < .001$, $d = 3.04$, and the posttest (69% vs. 85% correct), $t(28) = 2.86$, $p < .001$, $d = 1.45$. However, learning was again greater among those whose initial performance was relatively poor. Magnitude comparison accuracy improved for children with lower pretest magnitude comparison accuracy (54% vs. 69% correct), $t(15) = 5.37$, $p < .001$, $d = 1.46$, but not for children with higher pretest accuracy (84% vs. 85% correct).

Counting. No analysis was possible because 100% of children with pretest scores above the median counted perfectly.

Arithmetic. A median split on addition pretest performance led to identification of a group of 16 children with average absolute error of 2 or more ($M = 3.13$) and a group of 14 children with average absolute error of less than 2 ($M = 1.25$). Average absolute error varied with group, $F(1, 28) = 4.79$, $p < .05$, $\eta_p^2 = .15$; session, $F(1, 28) = 10.28$, $p < .01$, $\eta_p^2 = .27$; and the Group × Session interaction, $F(1, 28) = 10.97$, $p < .01$, $\eta_p^2 = .28$. Once again, learning was greater among children whose pretest performance was worse. Absolute error on all trials improved from pretest to posttest for children who generated larger errors on the pretest (mean error = 3.13 vs. 0.94), $t(15) = 5.23$, $p < .001$, $d = 1.81$, but not among those who generated smaller errors on the pretest (mean error = 1.25 vs. 1.29). The two groups differed on the pretest (mean error = 3.13 vs. 1.25), $t(28) = 4.48$, $p < .001$, $d = 1.64$, but not on the posttest (mean error = 0.94 vs. 1.29).

Might the ceiling effects that were so strong on the counting task also have made it impossible for children with pretest scores above the median to learn as much from the linear board game on the other four tasks? The answer appeared to be no. On the pretest, the percentage of children in the above median group whose scores were at ceiling was 0% on number line estimation, 0% on magnitude comparison, 18% on numeral identification, and 0% on arithmetic. On the posttest, the percentage of these children who were at ceiling was 0% on number line estimation, 6% on magnitude comparison, 35% on numeral identification, and 21% on arithmetic. Children whose pretest scores were above the median also produced far less learning than was possible. For example, on magnitude comparison, performance among these children improved only from 84% to 85% correct; on numeral identification, it improved only from 8.3 to 8.8 answers correct; and on the arithmetic task, children with below median pretest scores actually answered more accurately on the posttest. Thus, ceiling effects could not explain why children with more initial knowledge learned less than children with less initial knowledge.

Discussion

The results of this study were consistent with each of its main hypotheses. Playing the linear number board game led to considerably greater learning than playing the circular game or engaging in the numerical control activities. The advantages of playing the game with the linear rather than the circular board were specific to measures that reflected understanding of numerical magnitudes. Individual differences were quite stable from pretest to posttest within each condition, even in the linear board condition where substantial learning occurred. Probably most striking was the learning to learn effect in arithmetic: Children who earlier had played the linear board game learned more from subsequent practice and feedback on addition problems than children in the other two conditions. Thus, increasing understanding of numerical magnitudes is crucial not just for its direct effects but also because it improves children's subsequent learning of arithmetic. In this concluding section, we examine several implications of these findings.

Implications for Arithmetic Learning

The present findings suggest a different perspective on the processes involved in arithmetic learning than the typical one. Learning of answers to arithmetic problems has usually been viewed as a simple associative process (e.g., Campbell, 1991; Zbrodoff & Logan, 2005). Within some models, arithmetic performance depends on the frequency of past problem presentation, which is viewed as determining the strength of association between each problem and its correct answer (e.g., Ashcraft, 1992; Ashcraft & Christy, 1995). Within other models, the answers that learners generated in past efforts to solve a problem determine the associative strength between the problem and both correct and incorrect answers (Siegler & Shipley, 1995; Siegler & Shrager, 1984). Yet, other models emphasize priming of associations between problems and answers, interference between problems that share operands, and associations between pairs of operands and answers to other arithmetic operations (Campbell, 1987, 1991; Campbell & Graham, 1985).

The impact of numerical magnitude representations on arithmetic performance has received far less attention. Evidence documenting this influence has long been available, but it has been explained away as a byproduct of associative processes. For example, data from verification tasks indicate that people are slower to reject close misses (e.g., $4 + 6 = 12$) than errors that are farther in magnitude from the correct answer (e.g., $4 + 6 = 18$). Ashcraft and Stazyk (1981) explained such verification data by hypothesizing that addition answers are represented in an associative network, much like a multiplication table, and that interference from the correct answer lengthens the time to reject close misses. A different associative explanation of magnitude effects has been advanced for data from production tasks. Siegler (1988) found that close misses (e.g., $5 \times 2 = 12$) are retrieved more often than errors more distant in magnitude ($5 \times 2 = 18$). He noted that most errors generated by repeated addition, the main backup strategy in multiplication, are close in magnitude to the correct answer and argued that the close misses produced through repeated addition become associated with the problem, leading to their being retrieved and stated more often than other errors.

The present data in no way argue against the importance of associative processes in arithmetic. However, they do argue that understanding of magnitudes plays an additional role that cannot be easily explained by these associative accounts. Children in the three conditions of the present study had identical opportunities to associate arithmetic problems with their answers, yet children who previously had played the linear board game learned considerably more from the arithmetic experience than children in the other two conditions. What type of learning process would lead to this pattern?

Our analysis begins with an analogy to story comprehension: Magnitudes seem to play a role in learning arithmetic (and other numerical information) akin to the role of gist in learning the content of stories. Bransford and Franks's (1971) and Bransford and Johnson's (1973) classic studies demonstrate that understanding the gist of a story is essential for learning its content. The same passage can be remembered perfectly or not at all, depending on whether readers can extract its gist.

Understanding of numerical magnitudes, like understanding of gist, improves recall. There's a reason why newscasters and pundits often confuse numbers like 400 million and 400 billion but never numbers like 4 and 40. In the one case, they do not understand the numbers' magnitudes; in the other, they do. Similarly, understanding numerical magnitudes makes recalling answers to arithmetic problems more like recalling a set of related and meaningful facts than like recalling a list of unrelated words.

The analogy can be applied to understanding elementary school children's relatively good knowledge of whole numbers and their poor knowledge of fractions. No second grader would say that $3 + 3 = 3$, yet many sixth graders say that $1/3 + 1/3 = 2/6$ (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981). Similarly, no second grader would say that 274 is smaller than 83, but most fifth and sixth graders say that .274 is larger than .83 (Hecht, 1998; Hecht, Close, & Santisi, 2003; Kouba, Carpenter, & Swafford, 1989; Resnick & Omanson, 1987).

This analogy leads to the following question: Through what mechanism would knowledge of numerical magnitudes facilitate the learning of answers to arithmetic problems? One plausible hypothesis is that the mental number line serves as a retrieval structure that improves encoding, storage, and retrieval of numerical information by organizing the information around the numbers' magnitudes. Retrieval structures appear to be a key component of expert memory on such diverse tasks as remembering configurations of chess pieces (Chase & Simon, 1973), sequences of numbers (Chase & Ericsson, 1982), restaurant orders (Ericsson & Polson, 1988), and intermediate steps within multidigit mental multiplication (Staszewski, 1988). The ways in which retrieval structures enhance recall, recognition, and other forms of memory have been specified within computer simulations of expertise in computer programming, mental multiplication, and other areas (e.g., Adelson & Soloway, 1988; Richman, Staszewski, & Simon, 1995).

In the present context, playing the linear board game seems to have helped preschoolers form a retrieval structure for encoding, storing, and retrieving single-digit numbers. The clearest evidence of the effects on encoding came from the arithmetic error data. The posttest arithmetic errors of children who had played the linear board game were closer to the correct answer than their pretest errors had been, a pattern not shown by children in the other two conditions. This pattern was strongly indicative of enhanced encoding of numerical magnitudes during learning. Why else would children who had played the linear board game increasingly make close miss errors, whereas children in the other two conditions, who were given identical arithmetic experience, did not do the same? The fact that children who had played the linear board game also generated a greater number of correct answers to the arithmetic problems suggests that after receiving feedback on them, the children's storage and retrieval of correct answers also benefited from having played that game.

A child who uses a linear mental number line as a retrieval structure for numerical information is in a position to induce regularities among the answers to arithmetic problems. For example, in whole number addition, a child who accurately represents numerical magnitudes could learn that the sum is invariably greater than either addend, that pairs of large single-digit addends have answers closer to 18 than pairs of small or medium size single-digit addends, and that the sum is always closer to the larger addend than to the smaller one. Such knowledge considerably constrains the potential answers to single-digit arithmetic problems. It also makes possible generation of useful strategies. For example, noticing that the sum is always closer to the larger addend than to the smaller one is essential for generating the min strategy (adding by counting on from the larger addend). These results suggest that future models of arithmetic will benefit from including retrieval structures or other mechanisms that embody numerical magnitude representations.

The Representational Mapping Hypothesis

One major goal of this study was to test the representational mapping hypothesis, which states that the greater the transparency of the mapping between physical materials and desired internal representations, the greater the learning of the desired internal representation. In the present context, this hypothesis implied that a linear board game would lead to greater learning than a circular board game because the linear board more closely resembled the desired mental representation. The advantage was hypothesized to be specific to tasks that benefit from linear representations of numerical magnitude.

The data were consistent with this hypothesis. On the number line estimation task, linearity, slope, and accuracy were all greater on the posttest for those who played the game with the linear board than for those who played it with the circular board. On the magnitude comparison task, accuracy improved from pretest to posttest among children who played the game with the linear board but not among those who played the game with the circular board. On the arithmetic task, both percentage correct and quality of errors were higher following exposure to arithmetic problems and their sums for those who earlier had played the game with the linear board but not for those who had not. Also as predicted, numeral identification improved similarly from pretest to posttest for those who played either the linear or the circular board game.

This evidence does not imply that people cannot form circular representations of numerical magnitudes. When instructed to imagine numbers arranged clockwise around an analog clock face, adults appear to generate and rely on the requested circular representation (Bächtold et al., 1998). After such instructions, adults responded more quickly to small numbers with their right hand and more quickly to large numbers with their left hand, a pattern consistent with the requested clock face representation (and the opposite of the pattern seen in the absence of such instructions). Clearly, adults can form circular representations of numerical magnitudes if requested to do so; it seems likely that the same is true of children at some age.

Nonetheless, the present data suggest that not all representations of numerical magnitude are equally easy to learn. Nothing about the pretest data indicated that the preschoolers in this study had formed either a linear or a circular representation of numerical magnitudes prior to the experimental session. However, a single hour of experience playing the linear number board game appeared sufficient for children to form a linear representation and to use it to improve performance on numerical magnitude comparison, number line estimation, and arithmetic tasks. The same amount of experience playing the circular board game had much less effect on performance on these tasks.

These data indicate that just as research on mental representations can lead to predictions of the effects of physical materials on learning, data on learning can provide useful information about the relative difficulty of forming alternative mental representations. It does not appear to be happenstance that people form linear rather than circular representations of numerical magnitudes; linear representations are easier to learn.

The approach used in the present study may also prove useful in answering many other questions about mental representations. Consider three such questions regarding numerical representations. Are horizontally oriented representations of numerical magnitudes easier to learn than vertically oriented ones? Do children in our culture who cannot yet read find it easier to learn left-to-right representations than right-to-left ones? Is learning a logarithmic representation from playing a logarithmically spaced number board game easier than learning a linear representation from playing a linearly spaced number board game? Examining ease of learning is likely to prove useful in answering a wide range of questions about mental representations in other content domains as well.

The Wisdom of Culture—and Experimental Evidence

Linear number board games akin to *Chutes and Ladders* have a surprisingly long history. Similar games appear to have been played in India as early as the 2nd Century B.C. and in Southeast Asia from at least the 13th Century A.D. The South Asian version of the game, known as *Snakes and Ladders*, remains a popular activity in contemporary India, Pakistan, and Nepal (Parlett, 1999). A related game by the same name was introduced to England in 1892, and a variant of the British game was introduced to the United States in 1943. Entertainment and, in the case of the Asian version, spiritual enlightenment are the ostensive purposes of such games. However, if we can generalize from the present findings, playing them also improves young children's numerical cognition.

Many venerable cultural practices that involve children seem to fit this profile: Their ostensive purpose is not to convey cognitive skills, but they do. One example is nursery rhymes, including modern variants such as Dr. Seuss stories. The apparent purpose of these rhymes is to entertain children, to provide a setting for pleasant parent–child interactions, and to help children go to sleep. However, nursery rhymes also appear to promote phonemic awareness, which, in turn, promotes reading skill (National Reading Panel, 2000). Children's knowledge of nursery rhymes at age 3 predicts their later phonemic awareness and reading readiness, even when the mother's educational level and the child's IQ are statistically controlled (Maclean, Bryant, & Bradley, 1987). The minimal contrasts that often occur between words at the ends of lines of nursery rhymes (*horn* and *corn*, *muffet* and *tuffet*, *ham* and *am*) may help children to recognize that words are composed from separable sounds and to separate the individual sounds that are present within each syllable. These are useful skills for learning to read.

The relation between folk tales and problem solving provides another example. Chen, Mo, and Honomichl (2004) presented Chinese and U.S. college students with two problem-solving tasks. One problem could be solved in a way that paralleled a strategy used in a folktale well known in China but not in the United States. The other problem could be solved via a strategy like that used in *Hansel and Gretel*, a story well known in the United States but not in China, in which Gretel lays down shiny pebbles so that she and her brother can find their way home. Childhood exposure to these stories greatly influenced the college students' problem solving many years later. The problem whose solution paralleled that in the Chinese folktale was solved by 69% of the Chinese students but only by 8% of the American students. The story whose solution paralleled that in *Hansel and Gretel* was solved by 75% of American students but only by 25% of Chinese students. Students in each culture who explicitly recalled the relevant folktale were more likely to solve the problem than those who did not.

Cultural activities may promote cognitively useful values as well as specific skills and knowledge. One example involves Girl Scout cookie drives (Rogoff, 1995). Although the ostensive goal of such drives is to raise money for the troop, the activity also promotes a variety of values that seem useful for learning: realistic planning, accurate record keeping, precise calculation, and so forth. The Girl Scouts' record sheets—which include customers' names; the types, quantities, and prices of cookies ordered by each customer; advance payments; and sales and delivery dates—also convey how to display data in tabular form.

Although these examples suggest that common cultural practices are a useful source of hypotheses regarding means for helping children learn, rigorous experimental tests are essential for going beyond this stage. Many cultural practices that might plausibly be helpful for promoting learning prove not to be when carefully tested. For example, in the present study, the numerical activities condition included several common practices in homes, preschools, and childcare centers: counting from 1–10, counting objects, and naming numerals. These practices are probably useful over longer periods, but they did not promote understanding of numerical magnitudes as effectively as playing the linear board game. Similarly, our previous studies (Ramani & Siegler, 2008; Siegler & Ramani, 2008) used as a control condition a game modeled after *Candy Land*, a popular activity among U.S. preschoolers. This game includes features that could promote one to one correspondence, a skill hypothesized within Piagetian theory to be central to numerical understanding (e.g., Piaget, 1952). Nonetheless, it did not lead to learning of any of the numerical skills that were tested. Thus, although common cultural practices are a useful source of hypotheses about activities that might promote learning, rigorous experimental testing is necessary to establish whether they do in fact have the desired effect.

Practical Implications

The present findings add to an increasing body of literature indicating that efforts to improve the numerical understanding of preschoolers from low-income backgrounds can yield large, broad, and rapid improvements. The benefits of playing the linear number board game extend to a variety of aspects of early numerical understanding: knowledge of numerical magnitudes, counting, numeral identification, and arithmetic. All of these are foundational skills that contribute to later mathematics learning. It is especially encouraging that children whose pretest performance was in the lower half of the present sample learned more and thus caught up partially or completely to children whose performance was initially more advanced.

These findings converge with data on children's game playing outside of preschool. The greater the amount of experience that children have playing board games at their homes and at the homes of friends and relatives, the more proficient the children are at number line estimation, numerical magnitude comparison, counting, and numeral recognition (Ramani & Siegler, 2008). The same children's experience with card games and video games did not show similar correlations with their numerical capabilities. The linear numerical board game *Chutes and Ladders* was the second most commonly cited board game, and whether children had played it was significantly correlated with performance on all four numerical competencies. Consistent with the hypothesis that playing board games contributes to differences in numerical knowledge among children from different backgrounds, children from middle-income families reported playing far more board games (though fewer video games) than their low-income peers. These findings about game playing in the everyday environment, together with the present and previous lab-based findings, indicate that part of the gap between low-income and middle-income children's mathematical knowledge when they enter school is due to differing experiences playing board games, particularly linear number board games.

The linear number board game used in the present study has several advantages that recommend it for widespread use. It involves little, if any, expense; a board could easily be drawn on a piece of paper or cardboard, small household objects could be used as tokens, and a spinner or even a coin could be used to determine the number of spaces on each move. The game also requires little, if any, instruction for parents or preschool teachers who might want to use it. Even overworked parents could fit in a game or two before bedtime, given that each game takes about 3 min. Further, the game does not require much total investment of time to produce large gains; it produced its effects in the present and previous studies after roughly 1 hr of play.

This number board game is not the only approach that has been shown to be effective in improving young children's numerical understanding. Wilson, Revkin, Cohen, Cohen, and Dehaene (2006; Wilson, Dehaene, Dubois, & Fayol, 2008) have reported promising results with an adaptive software program designed to improve young children's number sense. Their program has shown promising results with 7- to 9-year-olds with mathematical difficulties (Wilson et al., 2006) and with kindergartners from low-income backgrounds (Wilson et al., 2008). Other experimental interventions have also produced improvements in young children's numerical knowledge (e.g., Malofeeva, Day, Saco, Young, & Ciancio, 2004).

More comprehensive curricula for improving low-income preschoolers' and kindergartners' mathematical knowledge have also shown large positive effects. Following participation in the *Number Worlds* curriculum in kindergarten, low-income children from the United States performed as well as the mean of age–grade peers from China and Japan on a test of computational skill that was administered at the end of first grade (Griffin & Case, 1999). Another curriculum, *Pre-K Mathematics*, led to kindergartners from low-income backgrounds having mathematical knowledge at the end of the program equivalent to that of middle-income peers who did not participate in it (Starkey et al., 2004). Similarly, the

Building Blocks curriculum (Clements & Sarama, 2007) led to preschoolers from low-income backgrounds making much greater progress than a control group in number, geometry, measurement, and recognition of patterns. These curricula include many specific activities, and it is impossible to know which activities had the greatest impact on particular types of knowledge. However, all three curricula demonstrate that a relatively modest amount of input goes a long way in improving low-income preschoolers' mathematical understanding.

These findings raise the question of why both laboratory and field-based interventions can produce such large and rapid positive effects on low-income preschoolers' mathematical knowledge. One explanation is that prior to the intervention, such children may have had few experiences where their attention was focused on mathematics. Observations of homes and preschools, as well as the self-reports of teachers and parents, suggest that the home and preschool environments provide children with relatively little experience where their attention is focused on mathematics, far less than literacy-oriented experience (Plewis, Mooney, & Creeser, 1990; Starkey & Klein, 2000; Tizard & Hughes, 1984; Tudge & Doucet, 2004; Tudge, Li, & Stanley, 2008). For example, in a carefully conducted observational study of children's exposure to explicitly mathematical activities in their own homes, other people's homes, and child care centers, a majority of children from working class backgrounds were observed engaging in mathematical play or mathematical lessons in 0 of 180 observations (Tudge & Doucet, 2004). Experiences that required a focus on numerical magnitudes appeared to be especially infrequent.

Interviews with parents in low-income families indicate that many believe that the primary responsibility for providing instruction in math belongs to preschool teachers (Holloway, Rambaud, Fuller, & Eggers-Pierola, 1995; Tudge & Doucet, 2004). However, as Tudge and Doucet (2004, p. 36) commented,

> If it is indeed correct that working-class parents look to preschool settings to provide children with mathematics experiences . . . our data suggest that they are mistaken—we found no evidence that children are more likely to be engaged in mathematical activities . . . in formal childcare centers than at home.

For this reason, and because of the large, varied, and rapid effects of early mathematical activity documented in this and other studies, multiplying the number of preschoolers who receive interventions of demonstrated effectiveness in promoting mathematics learning seems a goal worth pursuing.

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