Older and Younger Adults' Strategy Choices in Multiplication:
Testing Predictions of ASCM Using the Choice/No-Choice Method

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The choice/no-choice method provides a means of obtaining unbiased estimates of the performance characteristics of strategies. The three experiments in the study illustrate the method's usefulness for testing predictions of alternative models of strategy choice. The experiments focused on 20- and 70-year-olds' choices among mental calculation, use of a calculator, and use of pencil and paper as strategies for solving multidigit multiplication problems. As predicted by the Adaptive Strategy Choice Model (ASCM), (a) differences in the speed and accuracy yielded by the strategies were the strongest predictors of the frequency with which each strategy was chosen on a given problem; (b) features of problems exerted an additional independent influence; and (c) having a choice resulted in better performance than not having one. These results held true for both older and younger adults. Potential extensions of the choice/no-choice method and of ASCM are discussed.

Adaptive strategy choice is a pervasive characteristic of human cognition. People vary their choices in intuitively sensible ways in response to problem difficulty (DeLoache, 1984; Klayman, 1985), episodic success of strategies (Reber, 1987), changes in their own competence (Lemaire & Siegler, 1995), task instructions (Gardner & Rogoff, 1990), and demands on cognitive resources (Guttenberg, 1984). Adaptive choices, as indicated by positive correlations between frequency of use of a strategy on each problem and the speed and accuracy that the strategy produces on that problem, are present in such diverse populations as infants (Adolph, 1995), preschoolers (Geary & Bilingual-Dubree, 1989), school-age children (Koshmider & Ashcraft, 1991; Lemaire, Barrett, Fayol, & Abd, 1994), young adults (LeFevre, Sadesky, & Bisanz, 1996), and older adults (Geary & Wiley, 1991). They also are present in such diverse domains as locomoting up and down slopes (Adolph, 1995), solving arithmetic problems (Cooney, Swanson, & Ladd, 1988), recalling lists of numbers (McGilly & Siegler, 1990), and reading text (Brent & Routh, 1978; Goldman & Saul, 1991).

Formal models of strategy choice such as the adaptive strategy choice model (ASCM; Siegler & Shipley, 1995), ACT-R (Lovett & Anderson, in press), and the adaptive decision maker (Payne, Bettman, & Johnson, 1993) are based on the assumption that the performance yielded by strategies plays a large role in determining which strategies are chosen. For example, if one strategy generates faster performance than another on a problem and the two strategies are equally accurate, the first is likely to be chosen more often.

Within any model that posits that the performance generated by alternative strategies is among the key determinants of how often each is used (i.e., pretty much any imaginable model), unbiased estimates of the performance generated by the strategies is vital for deriving quantitative predictions regarding strategy choices. Unfortunately, obtaining such unbiased estimates is far from easy.

The most straightforward method for determining the speeds and accuracies of strategies, the method that has been used in all prior studies of strategy choices with which we are familiar, is the choice method. It involves presenting a set of problems, assessing strategy use on each one, and then calculating the mean speed and accuracy that accompany use of each strategy. This has been done in a great many experiments (e.g., Alibali & Goldin-Meadow, 1993; Geary, 1990; Geary & Brown, 1991; Geary & Bilingual-Dubree, 1989; LeFevre et al., 1996; Lovett & Anderson, in press; Siegler, 1986).

Unfortunately, the estimates of strategy characteristics generated by the choice method are biased by selection effects. These selection effects involve both the problems on which strategies are used and the people who use each strategy most often. For example, if a less accurate strategy
is used mainly on easy problems and a more accurate strategy is used primarily on difficult ones, the more accurate strategy may produce lower percentages of correct answers (due to its being used on harder problems) and thus appear to be less accurate. Similarly, if good students tend to rely on Strategy A and less good students tend to rely on Strategy B, Strategy A may appear to yield superior performance even if it is no better than the alternative (for example, because the good students are more careful in executing it).

Such selection effects are not merely a theoretical possibility, they are a common problem in strategy choice research. For example, on single-digit addition problems, elementary school children sometimes use retrieval and other times count from the larger addend. When kindergartners, first graders, and second graders are asked to solve simple addition problems in any way they want, retrieval is more accurate than counting from the larger addend, in the sense that it generates a higher percentage of correct answers: 94% versus 83% in Siegler (1987). However, the problems on which the children most often use retrieval are considerably easier than the ones on which they count. The problems on which retrieval predominates involve smaller addends and generally elicit fewer errors and shorter solution times regardless of which strategy children use. This makes it unclear whether retrieval generally produces more accurate performance or whether the higher percentage correct associated with it is due to the problems on which it is used.

Comparable biases occur in connection with different people using different strategies. Continuing with the example of arithmetic, more knowledgeable children tend to use retrieval more often and to answer more quickly and accurately when they do retrieve (Siegler, 1988).

Thus, whenever strategy choices are influenced by the speed and accuracy that the strategies yield, performance in choice situations are a biased reflection of each strategy’s speed and accuracy characteristics. In particular, the speed and accuracy of strategies that are used disproportionately on easy problems are overestimated, and the speed and accuracy of strategies that are used disproportionately on hard problems are underestimated.

In this article, we propose an alternative method—the choice/no-choice method—that avoids these biases. It is designed to yield the type of data needed for evaluating how strongly strategy choices vary with the performance yielded by the strategies and also to allow specification of other factors that influence strategy choices.

As suggested by the name, the choice/no-choice method requires testing each participant under two types of conditions: conditions in which participants can freely choose which strategy to use (the choice condition) and conditions in which they must use a given strategy on all problems (the no-choice condition). If there are two available strategies, there will be two subconditions in the no-choice condition: one subcondition in which participants must use one strategy on all trials and one subcondition in which participants must use the other strategy on all trials. Requiring participants to use a given strategy on all trials yields an unbiased estimate of the performance characteristics of that strategy (for the sample that generated the performance). Comparison of these unbiased estimates for the available strategies allows analysis of the degree to which differences in the performance that strategies yield are related to choices among the strategies. Comparison of performance in the choice and no-choice conditions indicates what, if any, benefit participants gain from having a choice.

In the present study, the utility of the choice/no-choice method is illustrated in three experiments. In Experiment 1, we examined its applicability to young adults’ choices of whether to solve multidigit multiplication problems by using mental arithmetic or by using a calculator. College students were required to use mental arithmetic to solve all items in one set of multiplication problems, were required to use a calculator to solve all items in a second set, and were free to choose either approach in a third set. In Experiment 2, we examined the usefulness of the choice/no-choice approach for studying developmental changes in strategy choices. This was done by presenting the same multiplication task to older adults and comparing their performance to that of the younger adults from Experiment 1. In Experiment 3, we examined whether the method could be generalized to situations involving more than two strategies. In particular, we tested its applicability to studying older adults’ strategy choices in multiplication when participants could choose mental arithmetic, a calculator, or pencil and paper.

These experiments yielded the type of data needed to pursue the two main substantive goals of the study: (a) to test predictions of a model of strategy choice (ASCM) regarding the influence of the performance yielded by strategies on choices among the strategies and (b) to establish similarities and differences in the strategy choices of younger and older adults.

The Adaptive Strategy Choice Model

Basic Assumptions

The Adaptive Strategy Choice Model (ASCM; pronounced “ask-em”) is a computer simulation of how strategy choices are made and how they change with age and experience (Siegler & Shipley, 1995). The model’s goal is to specify how people choose adaptively among strategies, in the sense of choosing each strategy most often under conditions in which it yields fast and accurate performance relative to available alternatives. The model’s basic organization is shown in Figure 1.

ASCM is based on three central assumptions. One is that information about the performance that has been generated by each strategy in the past is maintained in a database, which plays a large role in determining future strategy choices. The database is dynamically modified each time a strategy is used to solve a problem. Information at three levels of generality is included in the database: global data (speed and accuracy generated by each strategy, averaged over all problems), featural data (speed and accuracy of
each strategy on problems containing a particular structural feature, such as 10 as a multiplicand or a first addend of a given size), and local data (speed and accuracy of each strategy on particular problems).

A second critical assumption concerns determinants of strategy choices. Within ASCM, strategy choices are determined in large part by the performance that has been produced by each strategy in the past and by projections from that performance to the behavior that the strategy would be likely to yield if it were used on other problems. These projections are computed using regression analyses that estimate the speed and accuracy that a strategy would yield on a problem with given features, even if the strategy has never been used previously to solve that problem. This allows the model to make adaptive strategy choices in domains in which it has experience, even if it has never encountered the particular problem. It also is why unbiased assessment of the performance characteristics of each strategy is so critical to evaluating the model. ASCM is based on the belief that such performance characteristics are the largest determinant of the frequency with which each strategy is chosen. As ASCM’s database comes to include increasing amounts of data on the performance that each strategy yields on individual problems and on problems with particular features, the database increasingly closely approximates an unbiased estimate of the strategy’s effectiveness on the problems. Testing whether strategy choices correspond to the relative speed and accuracy yielded by available strategies thus requires unbiased estimates of each strategy’s performance characteristics.

ASCM’s third key assumption concerns how the choice mechanism operates. Strategies are chosen in proportion to their strength. The strength of each strategy primarily reflects its speed and accuracy relative to those of alternative strategies. A weighted combination of the three types of information—global, featural, and local—is used in computing the strengths. The weighting of each type of data reflects both the specificity of the data and the amount of available information at that level. Thus, local data are most heavily weighted when the problem has been frequently encountered; featural data are most heavily weighted when the particular problem has not been encountered very often but problems with relevant features have been; and global data are most heavily weighted when substantial databases do not exist for either the particular problem or for problems with relevant features.

ASCM was developed as a simulation of strategy choices in a domain, single-digit addition, in which people have substantial experience with all of the particular problems they encounter. However, it was designed to be equally applicable to domains in which people have little or no experience with many problems in the domain. One such domain, the one examined in the present study, is multidigit multiplication.

**ASCM’s Predictions**

One key prediction of ASCM concerns determinants of strategy choices. The percentage of use of a particular strategy on each problem tends to be highly correlated with two types of characteristics of the problem: structural features and performance characteristics. Structural features are inherent properties of the problem, such as the sum or product of the multiplicands in a multiplication problem. Performance characteristics are behaviors that the problem elicits, such as the accuracy or speed that is generated when solving the problem.

For the frequently encountered problems examined in previous studies, performance characteristics have correlated more highly with strategy choices than have structural features. For example, in young children’s single-digit addition, the percentage of errors elicited by a problem is more highly correlated with the percentage of retrieval on the problem than are such structural features as the problem’s sum, smaller addend, larger addend, first addend, or second addend (Geary & Brown, 1991; Geary, Brown, & Samaranayake, 1991; Geary & Burlingham-Dubree, 1989; Siegler, 1987; Siegler & Shrager, 1984). Similar results have been obtained in other domains in which the individual items are highly familiar, such as reading and spelling of familiar words (Siegler, 1986; 1988).

This result might be explained on the commonsense grounds that experience using different strategies on partic-
ular problems had taught children which strategy worked best on each one. But what of unfamiliar problems, ones on which people have had little if any direct experience? ASCM predicts that even on these problems, performance characteristics of the strategies should be better predictors than should any single structural feature. The reason is that experience with some problems allows children to project which strategies work best on other ones as well.

Attesting to this fact, Siegler and Shipley (1995) reported a run of ASCM in which the model was given experience with 10 of the 81 addition facts with addends 1–9. Then the model’s strategy choices on the other 71 problems were observed. Lacking direct experience with any of the 71 problems, the model still chose adaptively among the available strategies, in the sense of choosing each strategy most often on the problems where it yielded the greatest speed and accuracy relative to alternative strategies. The best predictors of its performance were the same as the best predictors of children’s performance. Presenting multidigit multiplication problems that people rarely, if ever, encounter allowed us to test whether the performance characteristics of the strategies (their relative speeds and accuracies) are also better predictors of people’s strategy choices on unfamiliar problems.

This prediction directly contrasts with that of several alternative models whose fundamental prediction is that the best single predictor of performance should be a structural feature, such as the product or product squared (Ashcraft, 1987; Geary & Widaman, 1987; Geary, Widaman, & Little, 1986). Within ASCM, associative strengths reflect several distinct sources of difficulty; performance characteristics, which within ASCM are viewed as reflecting all of these sources of difficulty, should therefore be better predictors of strategy choices than should any single structural feature.

A second prediction involves the role of featural information in strategy choices. Recall that within ASCM, information is represented at three levels: global, featural, and local. Featural information, like global information, allows extrapolation from previous experience with related problems. Past studies of multiplication have shown that speed and accuracy are influenced by such features as whether the problem includes 5, 1, or 0 as a multiplicand, and also by the odd–even status of the multiplicands (Krueger, 1986; Lemaire & Fayol, 1995; Siegler, 1988). ASCM predicts that the influence of these features should be especially large in situations such as this one, in which people have substantial experience with problems that have a specific feature but not with the particular problem. For example, even if $10 \times 23$ has never been encountered, previous experience with $N \times 10$ problems would be expected to increase the likelihood of using mental arithmetic to solve the problem, above and beyond what would be expected from the differences in speed and accuracy yielded by mental arithmetic and calculator use on such problems.

A third prediction of ASCM is that being able to choose among strategies should result in better performance than not being able to choose. This effect should be evident above and beyond the frequency with which each strategy is chosen on the total set of problems. The reason for this prediction is that relative effectiveness of strategies varies with the type of problem. Presumably, when people are free to choose strategies, they most often choose the strategy that works best on the particular problem for them.

The choice/no-choice methodology also allowed examination of several other issues suggested by ASCM, ones where the model raises questions rather than yielding specific predictions. One such issue is whether strategy choices are biased, as well as being adaptive. The fact that choices are finely calibrated to the relative speed and accuracy yielded by strategies does not preclude the choices also being biased toward one strategy or another. If choices are unbiased, two strategies that yield equal speed and accuracy should be equally likely to be chosen. As shown in Figure 2, the adaptiveness of strategy choices and their degree of bias are conceptually independent: Strategies can be adaptive and unbiased, adaptive and biased, nonadaptive and unbiased, or nonadaptive and biased. For example, Figure 2C illustrates a case in which choices are perfectly adaptive but also biased. They are perfectly adaptive in that the greater the speed of Strategy A relative to Strategy B, the more often participants choose Strategy A. However, the choices are also biased toward use of Strategy A, in that when there is no difference in relative speed, participants choose it more often than Strategy B. The no-choice condition indicates the performance that each strategy generates on each problem on the total set of problems and thus allows determination of whether, in the choice condition, participants choose strategies equally often on problems where they yield equivalent performance. Observing that people are biased in their strategy choices would indicate that rational models of strategy choice, such as ASCM and ACT-R, need to be supplemented to include factors beyond the speed and accuracy that strategies generate.

The choice/no-choice procedure also yields the type of precise data that are needed to evaluate the contributions of components of cognitive change to the overall changes that occur. ASCM distinguishes among four components of strategic change: introduction of new strategies, shifts in the relative frequency of use of existing strategies, increasingly efficient execution of each strategy, and increasingly adaptive choices among strategies. It is difficult if not impossible to cleanly separate these sources of change within standard choice procedures. Changes in the mean speed and accuracy of different strategies are confounded with changes in the problems on which each strategy is used and in the individuals who are using each one.

Presenting people of different ages the choice/no-choice method, in contrast, yields unconfounded estimates of the contribution of each source of change. In particular, comparison of performance at different ages in the no-choice conditions indicates the degree of improvement in execution of each strategy, unconfounded by changes in when and how often each strategy is used. The choice/no-choice method also allows examination of changes in the adaptiveness of strategy choices independent of changes in frequency of use of the strategies or changes in their speed and accuracy of execution. The reason is that adaptiveness of choices in the choice condition is evaluated relative to
Figure 2. Illustration of conceptual independence of adaptiveness and bias of strategy choices. Adaptiveness involves the strength of covariation between the frequency with which a strategy is used and the performance it yields relative to alternative strategies. Bias involves the tendency to choose a strategy more often than warranted by the performance it yields. Figure 2A illustrates performance that is both adaptive (the larger the advantage of Strategy A over Strategy B, the more often Strategy A is used) and unbiased (the two strategies are used equally often when performance yielded by them, as indexed by mean solution time, is equal). Figure 2D shows performance that is both nonadaptive (strategy choices do not vary with the relative performance yielded by the two strategies) and biased (Strategy A is used more often even when it does not yield better performance). Figure 2B and 2C illustrate how performance can be adaptive but biased (2B) or nonadaptive but unbiased (2C). RT = reaction time.

differences in speed and accuracy yielded by each strategy in that age group’s no-choice condition. The choice condition also allows examination of introduction of new strategies from one age to another and of changes with age in the relative frequency of strategies.

Development of Strategy Choices During Adulthood

As noted by Salthouse (1991; see also Craik & Salthouse, 1992), almost all research on cognitive change during adulthood has focused on changes in speed and accuracy. Pre-
rious research in the area of the present study, mental arithmetic, conforms well to Salthouse’s general observation. The most common finding of studies that have examined arithmetic performance among younger and older adults is that both speed and accuracy decrease with age (Allen, Ashcraft, & Weber, 1992; Birren, Allen, & Landau, 1954; Birren & Botwinick, 1951; Charness & Campbell, 1988; Geary, Frensch, & Wiley, 1993; Geary & Wiley, 1991; Salthouse & Coon, 1994). The two previous studies that have examined changes in strategy choices during adulthood (Geary, Frensch, & Wiley, 1993; Geary & Wiley, 1991) also indicate that the choices of both older and younger adults vary with structural measures of problem difficulty. However, none of the studies has indicated whether strategy choices vary with performance characteristics of the strategies nor whether these relations are stronger than the relations to structural features of the problems.

Two hypotheses have been proposed regarding sources of age-related changes in performance: the production deficiency hypothesis and the processing deficiency hypothesis (Salthouse, 1991). The production deficiency hypothesis states that age-related differences in performance reflect younger adults choosing strategies more adaptively than older ones. This view implies that in the present situation (a) the correlation between the percentage of use of each strategy on each problem and the difference in speed and accuracy yielded by the strategies on that problem should be higher for younger than for older adults and (b) the superiority of performance in the choice condition relative to the no-choice condition should be greater for younger than for older adults.

The processing deficiency hypothesis suggests that the choices of older and younger adults are equally adaptive, but that younger adults execute the strategies more quickly and accurately. In the present context, this hypothesis implies (a) the correlation between the percentage of use of each strategy on each problem and the difference in speed and accuracy yielded by the strategies on that problem should be equal for younger and older adults (b) the degree of superiority of performance in the choice condition relative to that in the no-choice condition should be equal for older and younger adults; and (c) substantial differences should be present between younger and older adults’ speed and accuracy.

No previous experiments have been conducted on the choices examined in the present study: mental arithmetic versus using a calculator in Experiments 1 and 2 and mental arithmetic versus using a calculator versus using pencil and paper in Experiment 3. However, these choices were of special interest in the context of a comparison of younger and older adults. Comparing these particular age groups on these particular strategy choices enabled us to examine how sociohistorical changes introduced by new technology influence the strategy choice process. Calculators have been widely available throughout the time that current college students were learning multiplication. In contrast, calculators didn’t exist when current older adults learned multiplication; they continued to be nonexistent, and then very rare, for years thereafter. Even today, a great many older adults never or almost never use calculators. The choice/no-choice method made it possible to see if these influences led to older adults showing a greater preference than younger adults for mental arithmetic, both in absolute terms and relative to what would be predicted by the speed and accuracy that each age group generated when using each approach.

Experiment 1

Method

Participants

Forty-eight undergraduates at Carnegie Mellon (10 men and 38 women) participated in partial fulfillment of course requirements. Their mean age was 20.6 years, with ages ranging from 18.1 to 25.4 years. The experimenters were a 28-year-old research assistant and a 30-year-old postdoctoral student (the second author).

Stimuli

The stimuli were 72 multiplication problems, presented in standard form (i.e., n × m). The problems were divided into three subsets of 24 problems each. Within each subset, there were four types of problems: 4N × 10 problems (e.g., 7 × 10), 4N × 10 problems (e.g., 10 × 16), 8N × NN problems (e.g., 7 × 17), and 8NN × NN problems (e.g., 16 × 13). On N × 10 and NN × 10 problems, the 10 could be either the first or second multiplicand.

To match the difficulty of the three subsets of problems, we equated mean product size, as well as the types of problems, across the sets. Mean products were 192, 196, and 191 for Subsets 1, 2, and 3, respectively. Product size was used as the matching variable because in past studies (e.g., Siegler, 1988), it usually has been the structural variable most highly correlated with speed and accuracy. Appendix A lists the problems in each subset.

The calculator was a Sharp EL-334H (Sharp Electronics Corp., Mahwah, NJ). It was considerably larger than most calculators, 15.53 cm long and 10.17 cm wide. It also had unusually large keys; for example, the keys for the 10 numbers were each 1.52 cm square, and the numbers and signs were printed in large type on them. The calculator was a relatively simple one, including only the 10 keys for the digits, the 4 main arithmetic-operation keys, an equal-to sign, 5 red-and-blue function keys, and 4 other keys (percent, decimal point, ±, and square root).

Experimental Conditions

Every participant solved problems under three conditions. In the choice condition, participants could use a calculator or mental arithmetic to solve the problem. In one no-choice condition, the calculator-required condition (CR), participants were required to use the calculator to generate an answer to each problem. In the other no-choice condition, the mental-arithmetic-required condition (MAR), participants were required to use mental arithmetic to solve the problems. Each subset of problems was seen by an equal number of participants under each of the three conditions. The subsets of problems were equivalent in the speed and accuracy they elicited in the no-choice conditions and in the percentage of
choices of mental arithmetic they elicited in the choice condition ($F_{S} < 1.02$).

Procedure

Before encountering the experimental problems, the students were presented the calculator and were shown how it worked. All were familiar with calculators, and none indicated a need to practice using it. All participants were told, "You are going to see problems such as $11 \times 5$ on the screen, and your task is to tell me the solution. You will have to solve three series of 24 multiplication problems each. To solve the first 24 problems, you can either use this calculator or find the solution in your head." Half the participants were then told that they would need to solve all problems in the second set of 24 in their heads and would need to solve all problems in the third set of 24 by using the calculator. The other half of participants were told the same thing in the reverse order.

The experimental problems were presented using numbers set in 48-point Times font in the center of a 16-inch computer screen controlled by a Macintosh Quadra 700 computer (Apple Computer Inc., Cupertino, CA). At the outset of each trial, the word ready appeared at the center of the screen for 500 ms. Then, the problem was displayed horizontally in the form "$w \times n" at the center of the screen. The symbols and numbers were separated by spaces the width of one character. Problems remained on the screen until the participant answered. Timing of each trial began when the problem appeared on the screen and ended when the experimenter pressed the space bar, the latter event occurring as soon as possible after the participant's response. The software (PsyScope [Cohen, MacWhinney, Flatt, & Provost, 1993]) collected data with 16-ms accuracy. However, for purposes of this experiment, times were coded to the nearest 0.1 s. On each trial, the experimenter typed the participant's answer into the computer. In the choice condition, the experimenter also typed in a code for the participant's strategy (mental arithmetic or calculator) on each trial.

Participants were randomly assigned to one of two presentation orders: choice–MAR–CR or choice–CR–MAR. The choice condition was always presented first so that choices would not be influenced by recency effects from just having used a given strategy on 24 consecutive trials. Participants were given short rest periods following completion of the first and second subsets of problems. Sessions lasted 30–45 minutes.

Results

Results are reported in two main parts. The first provides an overview of performance in the choice and no-choice conditions. The second describes tests of ASCM's predictions concerning determinants of strategy choices in the choice condition and the effects of having a choice on speed and accuracy. Unless otherwise noted, differences are significant to at least $p < .05$. Initial analyses indicated that there were no order effects either between conditions or within them. Participants who performed the conditions in the two orders did not differ in speed or accuracy in any of the three experiments. Strategy use also did not change from the first half to the second half of the choice condition in any of them. Therefore, the data were grouped across orders in further analyses.

Overview of Performance Under No-Choice Conditions

In the MAR condition, mental arithmetic was required on each problem; in the CR condition, the calculator was required on each one. Performance under these conditions yielded estimates of the speed and accuracy characteristics of the strategies that would not be biased by selective use of the strategies on different problems and by different people. To compare performance yielded by the strategies, speed and accuracy in the no-choice condition were analyzed using 2 (required strategy: mental arithmetic or calculator) $\times$ 2 (multiplicand of 10: present or absent) $\times$ 2 (digits in multiplicand: 1 $\times$ 2- or 2 $\times$ 2-digit problems) within-subject analyses of variance (ANOVAs).

Speed. As shown in Table 1, solution times were faster when problems had to be solved by using the calculator than when they had to be solved by using mental arithmetic (4.30 s vs. 7.30 s), $F(1, 47) = 18.86$; 1 $\times$ 2-digit problems were solved faster than 2 $\times$ 2-digit problems (4.00 s vs. 6.10 s), $F(1, 47) = 218.08$; and problems with 10 as a multiplicand were solved more quickly than problems without 10 as a multiplicand (2.80 s vs. 7.30 s), $F(1, 47) = 151.09$.

All two-way interactions also were significant: Required Strategy $\times$ Multiplicand of 10, $F(1, 47) = 121.12$; Required Strategy $\times$ Digits in Multiplicands, $F(1, 47) = 103.26$; and Multiplicand of Ten $\times$ Digits in Multiplicands, $F(1, 47) = 101.22$. The three-way interaction was also significant, $F(1, 47) = 130.62$. Scheffe post hoc tests, used in analyzing all interactions in these experiments, indicated that this three-way interaction was due to the relative speeds produced by mental arithmetic and use of the calculator varying with the type of problem. Mental arithmetic was faster on all problems that included 10 as a multiplicand; using the calculator was somewhat faster on $N \times N$ problems that did not include 10 as a multiplicand; and the calculator yielded much faster performance on $N \times N$ problems that did not include 10 as a multiplicand (Table 1).

Accuracy. Results on errors paralleled those on solution times. The calculator produced fewer errors than mental arithmetic (1% vs. 17%), $F(1, 47) = 109.48$. The 1 $\times$ 2-digit problems produced fewer errors than the 2 $\times$ 2-digit ones (4% vs. 10%), $F(1, 47) = 134.74$. Problems with 10 as a multiplicand produced fewer errors than problems that did not include 10 as a multiplicand (1% vs. 13%), $F(1, 47) = 55.87$.

All two-way interaction effects also were significant: Required Strategy $\times$ Multiplicand of 10, $F(1, 47) = 151.81$; Required Strategy $\times$ Digits in Multiplicand, $F(1, 47) = 32.51$; and Multiplicand of 10 $\times$ Digits in Multiplicand, $F(1, 47) = 39.46$. As with the solution times, these two-way interactions were qualified by a significant three-way interaction, $F(1, 47) = 47.63$. It reflected error rates being much larger than anywhere else on 2 $\times$ 2-digit problems that did not include a 10 and that needed to be solved by using mental arithmetic (Table 1).
Table 1
Younger Adults' Percentage of Use of Each Strategy, Mean Solution Time (s), and Percentage of Errors in Choice and No-Choice Conditions (Experiment 1)

<table>
<thead>
<tr>
<th>Problems and means</th>
<th>Choice</th>
<th>No-choice</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% use</td>
<td>Mean solution time</td>
</tr>
<tr>
<td>Structural features of the problem</td>
<td>MA</td>
<td>C</td>
</tr>
<tr>
<td>N x 10</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>NN x 10</td>
<td>95</td>
<td>5</td>
</tr>
<tr>
<td>N x NN</td>
<td>45</td>
<td>55</td>
</tr>
<tr>
<td>NN x NN</td>
<td>28</td>
<td>72</td>
</tr>
<tr>
<td>Strategy means</td>
<td>56</td>
<td>44</td>
</tr>
<tr>
<td>Overall means</td>
<td>4.1</td>
<td>4</td>
</tr>
</tbody>
</table>

Note. The strategy means and overall means are not simple averages of the means for the four types of problems. In the no-choice condition, they reflect the fact that there were twice as many items of the problem types without 10 as a multiplicand as of the problem types that did have a 10. In the choice condition, the means reflect that fact and the different frequencies of use of the strategies on each type of problem as well. MA = mental arithmetic; C = calculator.

Overview of Performance in the Choice Condition

Strategy choices. Choices were analyzed using a 2 (multiplicand of 10: present or absent) x 2 (digits in multiplicands: 1 x 2-digit or 2 x 2-digit problems) within-subject ANOVA. Unsurprisingly, the calculator was used much more often on problems that did not include 10 as a multiplicand (64% vs. 3%), F(1, 47) = 285.99. It also was used more often on 2 x 2- than on 1 x 2-digit problems (39% vs. 28%), F(1, 47) = 38.59. The interaction between the variables also was significant, F(1, 47) = 9.82. On problems with a multiplicand of 10, greater number of digits led to only slightly greater use of the calculator (5% vs. 0%), whereas on problems without 10 as a multiplicand, more digits led to a larger difference in percent use of the calculator (72% vs. 55%).

Speed. The relative speeds associated with the strategies were the opposite of those in the no-choice condition. Rather than mental arithmetic being associated with slower solution times than the calculator, it was associated with faster ones (2.6 s vs. 6.1 s), t(46) = 17.05.1 The reason was that when participants were free to choose, they used mental arithmetic primarily on problems that they could solve quickly in this way. This led to the solution times produced by mental arithmetic being much faster on those problems on which participants chose to use mental arithmetic in the choice condition (M = 2.6 s) than on the unbiased sample of problems on which they used the same strategy in the no-choice condition (M = 7.3 s).

Accuracy. Participants erred on a higher percentage of trials when they used mental arithmetic than when they used the calculator (6% vs. 2%), t(46) = 3.15). This difference in error rates was in the same direction as in the no-choice condition, but was considerably smaller than the 17% versus 1% difference in error rates in that condition. The smaller difference in error rates, like the reversal of the ordering of solution times, was attributable to mental arithmetic being used on easier problems in the choice condition.

Tests of ASCM's Predictions

To evaluate ASCM's predictions regarding the adaptiveness of strategy choices, we (a) tested whether the difference between performance yielded by the strategies on a given problem in the no-choice condition was a better predictor of strategy choices on that problem in the choice condition than were any of the structural variables; (b) determined whether structural features, in particular the presence of 10 as a multiplicand, influenced strategy choices on these relatively unfamiliar problems above and beyond the difference in performance yielded by the strategies; and (c) examined whether being able to choose a strategy resulted in greater speed and accuracy above and beyond the frequency of use of each strategy.

Predictors of strategy choices. To test the first two predictions, we ran a stepwise regression analysis of the percentage of use of the calculator on each of the 72 problems in the choice condition. We examined two types of predictors: structural variables and performance variables. The structural variables were each problem's first multiplicand, second multiplicand, product, number of digits in the multiplicands, and whether the problem included 10 as a multiplicand. The two performance variables that were used as predictors involved the difference between the

1 As in previous studies (e.g., Siegler, 1988), only items on which the relevant strategies were used by at least three participants are included in the statistical analyses. At least this many observations are needed to obtain stable estimates of mean speed and accuracy on the problem.
mean RT on each problem under MAR and CR conditions and the difference between the percentage of errors on each problem under MAR and CR conditions. ASCM's basic assumption was that the greater the relative speed and accuracy of mental arithmetic on each problem (compared with those yielded by the calculator) the more often that mental arithmetic would be chosen. Note that the data for these two performance variables came from the no-choice condition, whereas the dependent variable that they were predicting, the percentage of use of the calculator, reflected performance in the choice condition.

Three variables accounted for significant independent variance in the percentage of use of the calculator on each problem: (a) the difference between solution times on the problem under MAR and CR conditions \( (R^2 = .72) \); (b) whether 10 was a multiplicand (partial \( R^2 = .14 \)); and (c) the percentage of the difference between the percentage of errors on the problem under MAR and CR conditions (partial \( R^2 = .01 \)). Together, the three variables accounted for 87% of the variance in the percentage of use of the calculator on the 72 problems.²

Consistent with ASCM's prediction regarding the importance of featural information in strategy selection, the calculator was used more often on problems with 10 as a multiplicand than would have been expected solely from the performance yielded by the strategies. Considered by itself, this variable accounted for 63% of the variance in the percentage of choices of the calculator on each problem. Although this was a substantial percentage of variance, differences in the speeds of execution of the two strategies accounted for an additional 23% of variance beyond that which could be accounted for by presence of 10 as a multiplicand. Thus, the close relation between strategy choices and the speeds yielded by the strategies was not attributable to the undergraduates relying on a rule of the form "use the calculator when there isn't a 10, use mental arithmetic when there is one."

**Bias.** Within the original regression analysis of strategy choices on all 72 problems, the intercept was significantly greater than 0. This was true when difference in solution times yielded by the strategies was entered as the only predictor of the percentage of use of the calculator. It also was true when differences in both solution times and errors were entered as predictors. The finding indicated that the undergraduates were somewhat biased toward use of mental arithmetic. An unbiased selection process would choose mental arithmetic and the calculator equally often when there were no differences between them in speed, accuracy, or both. In fact, on the problems where the difference between mean solution times under MAR and CR conditions was about 0 (±1 s), participants used mental arithmetic on 58% of trials. Not until MAR times were 1.5 s (±1 s) longer than CR times were the two strategies chosen equally often.

To test whether the bias was statistically significant, we identified the problems on which speed of performance was almost identical for the two strategies (±1 s) and compared the percentage of use of mental arithmetic on each one to the 50% expected from consideration of the speed yielded by the two strategies. The percentage of use of mental arithmetic on these 11 problems was greater than 50%, \( t(10) = 3.02 \), thus indicating that strategy choices were somewhat biased toward mental arithmetic.

**Effects on solution times of having a choice.** ASCM also predicted that performance would be faster and more accurate in the choice than in the no-choice condition. This could come about for two reasons: because people used the faster strategy on a higher percentage of problems in the choice condition, or because they assigned strategies to problems in adaptive ways. The latter was of greater interest in the present context. Therefore, to test whether strategy choices were adaptive, above and beyond the frequency with which each strategy was used in the set as a whole, we statistically controlled for overall use of each strategy. To do this, we first determined the percentage of use of mental arithmetic in the choice condition: 56%. Then we used this percentage to project what the mean speed and accuracy in the no-choice condition would have been given comparable overall strategy use but with no selective assignment of strategies to individual problems. Specifically, each participant's projected solution time for each problem in the no-choice condition was \( .56 \) (mean solution time under MAR conditions for that problem) \( \times 44 \) (mean solution time under CR conditions for that problem). This was equivalent to comparing performance under choice conditions to performance that would have arisen from using mental arithmetic on each problem on 56% of trials and using the calculator on 44%.

As predicted, performance was faster in the choice condition (Table 2). A 2 (condition: choice or no-choice) \( \times 2 \) (multiplicand of 10: present or absent) ANOVA indicated that participants were faster when they had a choice, \( F(1, 47) = 80.18 \). They also were faster on problems that included 10 as a multiplicand, \( F(1, 47) = 320.39 \). A significant Condition \( \times \) Multiplicand of 10 interaction, \( F(1, 47) = 10.63 \), was present, due to differences between the choice and no-choice conditions being larger on the problems that did not include 10 as a multiplicand. Thus, having a choice was beneficial on both types of problems, and the benefits were higher on the more difficult problems.

Although the differences between performance in the choice and no-choice conditions were largest on the most difficult problems, choice led to faster performance on each of the four kinds of problems than if participants had used the overall distribution of strategies (56% mental arithmetic, 44% calculator) on each problem. For \( N \times 10 \) problems, the times for choice and no-choice conditions were 1.4 and 2.3 s respectively; for \( NN \times 10 \) problems, 2.1 and 3.0 s; for \( N \times NN \) problems, 5.0 and 5.7 s; and for \( NN \times NN \) problems, 5.6 and 9.7 s.

**Effects on accuracy of having a choice.** The parallel analysis was conducted on the percentage of errors on each problem.

² Although number of digits exercised a significant effect in the ANOVAs, it did not do so in the regression analyses. The reason was that all of the variance captured by this variable was also captured by the difference in solution times, and the latter variable was considerably more highly correlated with the percentage of use of mental arithmetic on each problem.
Table 2
Younger Adults' Speed and Accuracy Under Choice and No-Choice Conditions on Problems With and Without a Multiplicand of 10 (Experiment 1)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Mean RT</th>
<th>% errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplicand of 10</td>
<td>1.9</td>
<td>1</td>
</tr>
<tr>
<td>No multiplicand of 10</td>
<td>5.2</td>
<td>6</td>
</tr>
<tr>
<td>No-choice</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplicand of 10</td>
<td>2.8</td>
<td>1</td>
</tr>
<tr>
<td>No multiplicand of 10</td>
<td>7.2</td>
<td>14</td>
</tr>
</tbody>
</table>

Note. Data in the no-choice condition are projected by multiplying the mean speed and accuracy of each strategy in the no-choice condition by the overall proportion of use of that strategy in the choice condition. RT = reaction time.

...problem. Again as predicted, error rates were lower when participants could choose their strategy, \(F(1, 47) = 44.94\). Accuracy was also higher on problems with 10 as a multiplicand, \(F(1, 47) = 108.33\). The significant interaction between the two variables, \(F(1, 47) = 34.46\), was due to accuracy under the two conditions differing only when the problem did not include 10 as a multiplicand. The percentage of errors for the four types of problems in the choice and no-choice conditions were 1% and 0%, respectively, on \(N \times 10\) problems, 0% and 1% on \(NN \times 10\) problems, 5% and 8% on \(N \times NN\) problems, and 8% and 21% on \(NN \times NN\) problems.

Summary

The results of Experiment 1 illustrated the value of the choice/no-choice design. Data from the usual choice condition would have led to the conclusion that mental arithmetic was the faster strategy. However, data from the no-choice condition showed that this finding was entirely due to selection effects. When each strategy had to be used equally often on each problem (the no-choice condition), the speeds differed significantly in the opposite direction. Use of the calculator yielded more accurate performance in both conditions, but the amount of difference was far greater in the no-choice condition. Thus, examining performance under no-choice conditions seems essential for obtaining accurate assessments of the speed and accuracy characteristics of each strategy.

The Experiment 1 results also supported the three predictions of ASCM that were tested. The difference in speed yielded by the strategies on each problem in the no-choice condition was the best predictor of strategy choices on that problem in the choice condition. Featureal information (a multiplicant of 10) accounted for significant additional variance in which strategy was chosen. Finally, having a choice increased speed on all types of problems and accuracy on problems without 10 as a multiplicand.

The findings also provided evidence for a somewhat counterintuitive assumption of ASCM: that people have a well-calibrated sense of the time that alternative strategies take and that they use this information to guide their strategy choices. The greater the relative speed of each strategy, the more often the strategy was chosen. Relative speeds of the strategies was the best predictor of the undergraduates' choices and a very good predictor in absolute terms, accounting for 72% of the variance in strategy choices on the 72 problems. The strength of this predictor suggests that, as ASCM assumes, people can reliably estimate the relative amounts of time that strategies take to execute and that they base strategy choices in part on this information.

Experiment 2

ASCM does not predict any limits to its own generality. However, all previous tests of it have been conducted with children and young adults. There were reasons to suspect that its predictions might fare less well in describing the choices of older adults. Older adults have far less experience with calculators than do younger adults. In the popular stereotype, they are reluctant to use, and are sometimes afraid of, newfangled gadgets.

This conception of older adults raised several specific questions. Do they use calculators less often than their performance with them, relative to mental arithmetic, predicts? Does their relative frequency of use of them on different problems vary less strongly with differences in performance between strategies than that of younger adults? Do they base strategy choices more heavily on the relatively superficial feature of presence of 10 as a multiplicand? Are they more biased toward use of mental arithmetic when the performance that they generate by using the calculator is equally good? These questions were addressed in Experiment 2.

Method

Participants

Fifty-one older adults (7 men and 44 women) from a senior citizens’ center were paid $5 each to participate in Experiment 2. Their mean age was 66 years (range = 61 to 73). The data of 3 of the 51 participants were excluded from the analyses, because of excessively high error rates: over 40%. All participants were asked what their occupation had been when they worked. Their answers indicated a wide and representative range: housewife, mail carrier, bookkeeper, physicist, cashier, secretary, maintenance worker, nurse, and so on. The mean score of the occupations on the Duncan Socioeconomic Index (SEI; Hauser & Featherman, 1977) was 44.3, indicating that on average, the occupations were of moderate socioeconomic status.

Apparatus, Materials, and Procedure

The apparatus, materials, and procedure were identical to those used in Experiment 1. At the beginning of the experiment, participants were told they would be solving some arithmetic problems either in their head or with a calculator. To familiarize the older adults with the calculator, the experimenter first demonstrated how the problem \(11 \times 18\) could be solved on it. They were asked to solve another problem on their own. For the few who
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could not solve that problem, the demonstrations were repeated until they could use the calculator.

Results

Results are reported in three sections: an overview of performance under choice and no-choice conditions, tests of ASCM’s predictions, and comparisons of younger and older adults’ performance.

Overview of Performance in No-Choice Condition

The same type of 2 (required strategy) × 2 (multiplicand of 10) × 2 (digits in multiplicand) ANOVAs of speed and accuracy were performed as in Experiment 1.

Speed. As shown in Table 3, older adults, like younger ones, solved 1 × 2-digit problems faster than 2 × 2-digit problems (7.2 s vs. 10.7 s), F(1, 47) = 80.77. They also solved problems with a multiplicand of 10 faster than problems without one (6.5 s vs. 11.4 s), F(1, 47) = 107.83. Also like the younger adults, the old ones tended to be faster when they used the calculator than when they used mental arithmetic (8.4 s vs. 11.1 s). However, this difference did not reach statistical significance, F(1, 47) = 2.24, p = .14.

All two-way interactions also were significant: Required Strategy × Multiplicand of 10, F(1, 47) = 110.85; Required Strategy × Digits in Multiplicands, F(1, 47) = 45.65; and Multiplicand of 10 × Digits in Multiplicands, F(1, 47) = 47.15. The interaction among the three variables also was significant, F(1, 47) = 40.69. Solution times generated using mental arithmetic were faster than those generated using a calculator on problems that included 10 as a multiplicand, equal in speed to those generated by using the calculator on N × NN problems that did not include 10 as a multiplicand, and slower than those generated by using a calculator on NN × NN problems that did not include 10 as a multiplicand.

Accuracy. The older adults solved the 1 × 2-digit problems more accurately than the 2 × 2-digit ones (7% vs. 16% errors), F(1, 47) = 71.74. They also were more accurate on problems that included a multiplicand of 10 (4% vs. 18% errors), F(1, 47) = 112.4, and more accurate with the calculator than with mental arithmetic (2% vs. 25% errors), F(1, 47) = 136.75.

All two-way interactions also were significant: Required Strategy × Digits in Multiplicands, F(1, 47) = 75.08; Digits in Multiplicands × Multiplicand of 10, F(1, 47) = 19.52; and Required Strategy × Multiplicand of 10, F(1, 47) = 86.5. The three-way interaction also was significant F(1, 47) = 18.29. This interaction was attributable to errors showing entirely different patterns under CR than under MAR conditions. Under CR conditions, errors were infrequent on all four types of problems. Under MAR conditions, errors were somewhat more frequent on 2 × 2-digit than on 1 × 2-digit problems with 10 as a multiplicand, and much more frequent on 2 × 2-digit than on 1 × 2-digit problems without 10 as a multiplicand.

Overview of Performance in the Choice Condition

Strategy choices. The same types of analyses as conducted in Experiment 1 indicated that older adults, like undergraduates, used the calculator much more often on problems without 10 as a multiplicand (59% vs. 8%), F(1, 47) = 254.38. They also used the calculator more frequently on 2 × 2-digit than on 1 × 2-digit problems (45% vs. 22%), F(1, 47) = 38.59. The interaction between the two variables also was significant, F(1, 47) = 9.87. As shown in Table 3, the calculator was chosen more often on 2 × 2-digit problems, both with and without 10 as a multiplicand, but the difference was considerably greater on the problems without a 10 (74% vs. 43%) than on problems with a 10 (15% vs. 1%).

Speed. Unlike in the no-choice condition, performance in the choice condition was faster on trials where mental arithmetic was used (5.8 s vs. 12.2 s), t(47) = 9.20. The reason was the same as with the younger adults; mental arithmetic was used disproportionately often on the easier problems.

Table 3

<table>
<thead>
<tr>
<th>Problems and means</th>
<th>% use</th>
<th>Mean solution time</th>
<th>% errors</th>
<th>Mean solution time</th>
<th>% errors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MA</td>
<td>C</td>
<td></td>
<td>MA</td>
<td>C</td>
</tr>
<tr>
<td>Structural features of the problem</td>
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</tr>
<tr>
<td>N × 10</td>
<td>99</td>
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<td>3.8</td>
<td>7.5</td>
<td>2</td>
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<tr>
<td>NN × 10</td>
<td>85</td>
<td>15</td>
<td>4.8</td>
<td>13.2</td>
<td>7</td>
</tr>
<tr>
<td>N × NN</td>
<td>57</td>
<td>43</td>
<td>6.7</td>
<td>12.3</td>
<td>14</td>
</tr>
<tr>
<td>NN × NN</td>
<td>26</td>
<td>74</td>
<td>9.5</td>
<td>12.1</td>
<td>35</td>
</tr>
<tr>
<td>Strategy means</td>
<td>59</td>
<td>41</td>
<td>5.8</td>
<td>12.2</td>
<td>12</td>
</tr>
<tr>
<td>Overall means</td>
<td></td>
<td></td>
<td></td>
<td>8.4</td>
<td></td>
</tr>
</tbody>
</table>

Note. MA = mental arithmetic; C = calculator.
Accuracy. In the choice condition, as in the no-choice condition, mental arithmetic produced more errors than using the calculator (12% vs. 1%), t(47) = 17.12. Also as with the younger adults, the difference in the percentage of errors produced by the two strategies was consistently smaller in the choice condition than in the no-choice condition.

Tests of ASCM’s Predictions

Predictors of strategy choices. The predictions regarding older adults’ strategy choices were tested by using a regression analysis like that in Experiment 1. Three factors proved to contribute independently to the percentage of variance accounted for (a) the difference between mean solution times on each problem under MAR and CR conditions ($R^2 = .61$); (b) whether one of the multiplicands was 10 ($R^2 = .18$); and (c) number of digits in the multiplicands ($R^2 = .04$). The three variables together accounted for 83% of the variance in the percentage of use of the calculator on the 72 problems. Thus, as with younger adults, the difference on each problem between the performance yielded by the two strategies was the best predictor of strategy choices on that problem. Also as with younger adults, featural information (presence of 10 as a multiplicand) influenced the strategy choices.

Because it seemed likely that older adults were less experienced with calculators than the undergraduates, it seemed especially plausible that they would base their choices largely on the approach “if there’s a multiplicand of 10, use mental arithmetic; otherwise, use the calculator.” As with the undergraduates, presence of a 10 accounted for a substantial percentage of variance in the percentage of choice of the calculator (58%). Also as with the undergraduates, however, differences in the speeds of execution of the two strategies also accounted for substantial variance (21%) beyond that which could be accounted for by presence of 10 as a multiplicand. Thus, the close relation between strategy choices and the speeds yielded by the strategies once more was not attributable to reliance on presence or absence of a multiplicand of 10.

Bias. Strategy choices of older adults showed the same type of bias as those of undergraduates. After differences in speed alone or in speed and accuracy yielded by the strategies had been entered into the regression equation, the intercept differed significantly from 0. On problems where the difference between mean solution times under MAR and CR conditions was $0 \pm 1$ s, the older adults used mental arithmetic on 59% of trials.

As in Experiment 1, we tested whether the bias was statistically significant by identifying the problems on which speed of performance was almost identical for the two strategies ($\pm 1$ s) and comparing the percentage of use of mental arithmetic on each one to the 50% expected from consideration of the speed yielded by the two strategies. As with the undergraduates, the older adults’ use of mental arithmetic was greater than 50%, t(11) = 3.27, indicating that their strategy choices also were biased toward mental arithmetic. Participants chose each strategy on 50% of trials on the set of problems where the difference in mean solution times was 1.6 s ($\pm 1$ s).

Effects on solution times of having a choice. To test the prediction that being able to choose a strategy on each problem would result in faster solution times, we conducted the same type of ANOVA as in Experiment 1. Again as predicted, performance was faster in the choice than in the no-choice condition, $F(1, 47) = 13.58$. It also was faster on problems with 10 as a multiplicand, $F(1, 47) = 182.25$. The two variables did not interact. Thus, being able to choose their strategy enabled older adults to be faster on all types of problems that were presented (Table 4).

Effects on accuracy of having a choice. The parallel analysis of the percentage correct on each problem indicated that accuracy was higher in the choice condition than in the no-choice condition, $F(1, 47) = 39.28$. Accuracy was also higher on problems that had 10 as a multiplicand, $F(1, 47) = 71.33$. Finally, a significant interaction between the two variables, $F(1, 47) = 28.97$, showed that choice led to greater accuracy only on problems without 10 as a multiplicand. On problems with 10 as a multiplicand, accuracy was equally high in both conditions.

Comparison of Performance of Younger and Older Adults

The basic phenomena of younger and older adults’ strategy choices were identical. The best predictor of strategy choices was the difference in solution times yielded by the two strategies on each problem. This was a powerful predictor in absolute terms, as well as the best predictor in relative terms. Presence of 10 as a multiplicand also influenced choices. Choices for both groups were slightly biased toward use of mental arithmetic relative to what would be expected from the performance characteristics of each strategy. These parallels suggest that age-related differences in performance are due to differences in efficiency of strategy execution rather than in the choice process.

As a first test of this interpretation, we examined differences between younger and older adults’ performance in the no-choice condition, which guaranteed that the two strategies would be used equally often overall, and equally often

| Table 4 |

Older Adults’ Speed and Accuracy Under Choice and No-Choice Conditions on Problems With and Without a Multiplicand of 10 (Experiment 2)  

<table>
<thead>
<tr>
<th>Condition</th>
<th>Mean RT</th>
<th>% errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplicand of 10</td>
<td>4.9</td>
<td>4</td>
</tr>
<tr>
<td>No multiplicand of 10</td>
<td>10.2</td>
<td>9</td>
</tr>
<tr>
<td>No-choice</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplicand of 10</td>
<td>6.1</td>
<td>5</td>
</tr>
<tr>
<td>No multiplicand of 10</td>
<td>11.6</td>
<td>20</td>
</tr>
</tbody>
</table>

Note. RT = reaction time.
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on each problem, by both age groups. We ran two ANOVAs, one on solution times and one on the percentage correct, that were of identical form: 2 (age: older or younger adult) × 2 (condition: MAR or CR), × 2 (multiplicand of 10: present or absent) × 2 (digits in multiplicand: 1 × 2- or 2 × 2-digit problems). To avoid redundancy with the analyses reported in previous sections, only findings involving the age variable are reported.

**Age-related differences in solution times.** Younger adults were much faster than older adults (5.8 s vs. 9.8 s), F(1, 94) = 56.37. As is evident in comparing Tables 1 and 3, this greater speed was present both when the calculator was required on all problems (4.3 s vs. 8.4 s for younger and older adults) and when mental arithmetic was required (7.3 s vs. 11.1 s). Performance of both age groups was faster when participants used mental arithmetic on the problems with 10 as a multiplicand and faster when they used the calculator on 2 × 2-digit problems that did not include 10 as a multiplicand. On 1 × 2-digit problems, younger adults were faster when they used a calculator, whereas older adults’ times were similar regardless of whether they solved the problems by using mental arithmetic or the calculator.

**Age-related differences in accuracy.** Younger adults were also more accurate than older adults (9% vs. 14% errors). This advantage was greatest in the MAR condition. It held for all four types of problems.

**Age-related differences in strategy choices.** To compare strategy choices at the two ages, a 2 (age: older or younger adult) × 2 (multiplicand of 10: present or absent) × 2 (digits in multiplicand: 1 × 2- or 2 × 2-digit problems) was run on the percentage of use of the calculator in the choice condition. The most striking result was that there was no significant difference in the frequency with which the calculator was chosen: 56% by younger adults and 59% by older ones. The pattern of use did differ somewhat; older adults used the calculator more often than younger adults on NN × 10 problems (15% vs. 5%), whereas younger adults used it more often than older ones on N × NN problems (55% vs. 43%).

We also compared the adaptiveness of the two age groups’ strategy choices. In particular, we compared the correlations between the percentage of use of the calculator in the choice condition on each problem and the difference in performance generated by the two strategies in the no-choice condition on that problem. The correlations were of similarly high magnitude and did not differ significantly. For differences in speed on each problem, the correlation was r = .85 for younger adults and r = .78 for older ones. For differences in accuracy, the correlations were r = .74 for older adults and r = .70 for younger ones.

**Summary**

The Experiment 2 findings with older adults again illustrated the value to the choice/no-choice design. Although solution times produced by use of mental arithmetic were considerably faster in the choice condition, performance in the no-choice condition revealed that this was entirely due to selective use of the strategies. When the two strategies had to be used equally often on all problems and by all participants (the no-choice condition), performance was faster using the calculator than using mental arithmetic.

The results of Experiment 2 also indicated that ASCM’s predictions generalized to a considerably older population than had been studied previously, one whose members had much less experience making the relevant strategy choice. All three of its predictions were as accurate for the older population as they had been for the younger one.

Comparisons of performance in earlier and later adulthood allowed us to specify the locus of age-related changes. Strategy choices were highly similar between the undergraduates and the older adults. The similarities included how often each strategy was used, the best predictor of the choices, the strength of the predictive relation, and the degree of bias in the choices. The strategy choices of older adults, like those of younger ones, were most influenced by the relative amounts of time that the strategies required, indicating that both groups could estimate quite accurately the relative times required for execution of the strategies. Where older and younger populations differed was in absolute levels of speed and accuracy when they used a given strategy.

**Experiment 3**

Experiments 1 and 2 demonstrated that the choice/no-choice method could be applied to cases in which people choose between two strategies. Experiment 3 was designed to illustrate its applicability to situations with more than two available strategies. As in Experiment 2, older adults were presented multiplication problems, but this time they were given a choice among three strategies: mental arithmetic, a calculator, or pencil and paper. Then they were presented three additional sets of problem. They were required to solve one by mental arithmetic (MAR), one through use of a calculator (CR), and one through use of pencil and paper (PPR).

Generalizing the findings to choices among three strategies was of interest for two distinct reasons. First, most everyday problems can be solved by more than two strategies; thus the three-choice situation was more representative of the population of choices that might be studied using the choice/no-choice approach. Second, extending the method to situations with more than two choices necessitated establishing the usefulness of a measure of strategy use other than absolute probability of use. The reason is that in such multistrategy situations, a given strategy may lose competitions to one strategy for one reason and to another strategy for another. As described below, conditionalizing probability of strategy use on faster strategies not being used provides a useful measure for such multistrategy situations.

The decision to examine choices of pencil and paper when a calculator was present was based on one of us seeing his mother balance her checkbook with pencil and paper even though a calculator was readily available. If she did, perhaps many other older adults also did. Her approach
differed from that of undergraduates. In a pilot study, we presented 10 Carnegie Mellon undergraduates the same problems with the same three choices as were presented to older adults in Experiment 3. The undergraduates used pencil and paper on 0% of trials. Because older adults have long experience using pencil and paper, however, and far less experience with calculators, we suspected that pencil and paper would be a more attractive option for them.

Another goal of Experiment 3 was to examine sources of individual differences in older adults' strategy choices. We presented a questionnaire asking about previous experience with calculators, attitudes toward math, experience with math, and so on. This information allowed us to examine separately the strategy choices of a subset of older adults of special interest: those who had no preexperimental experience with calculators.

**Method**

**Participants**

Sixty-five older adults (2 men and 63 women) from another senior citizens' center were paid $5 apiece to participate in Experiment 3. Their mean age was 75 years (range = 60–95 years). Five participants erred on more than 40% of trials; their data were excluded from the analyses.

**Stimuli**

A new set of 72 problems was created and divided into four subsets of 18 problems. Each subset included 3 $N \times 10$ problems, 3 $NN \times 10$ problems, 6 $N \times NN$ problems, and 6 $NN \times NN$ problems. Within each problem type, items were matched for product size. Appendix B lists the items in each subset.

**Procedure and Experimental Conditions**

All participants first participated in the choice condition, in which they could solve problems through any of the three strategies. Then they participated in the three no-choice conditions, which were presented in counterbalanced order. The 60 participants were divided into 4 groups of 15 subjects each, so that each problem was seen by an equal number of people in each condition. The procedure was exactly the same as that used in Experiments 1 and 2, with one exception. Before the experimental trials, participants were asked to solve nine problems (similar but not identical to those used in Experiment 3), three with each strategy, so that they would be familiar with all of them.

At the end of the session, participants were presented a questionnaire regarding their background. The questions focused on demographics (age and former occupation), experience with calculators ("How often have you used a calculator: Never, a few times in my life, a few times a year, once a month, twice a month, once a week, twice a day, every day"), liking for each of the three strategies ("1 = not at all much, 7 = very much"), confidence in answers generated by each of the three strategies (same scale), and liking of math (same scale). The demographic data indicated that they had occupied a wide range of occupations: bookkeeper, tavern owner, seamstress, housewife, salesclerk, statistician, secretary, teacher, hairdresser, and so on. Their mean score on the Duncan SEI was 45.8, indicating that their professions, on average, had been of moderate socioeconomic status.

**Results**

**Overview of Performance in No-Choice Condition**

The same type of Required Strategy $\times$ Digits in Multiplicand ANOVA was conducted as in Experiments 1 and 2. The only difference was that in Experiment 3, there were three required conditions: mental arithmetic required (MAR), calculator required (CR), and pencil and paper required (PPR).

**Speed.** As can be seen in Table 5, the older adults solved $1 \times 2$-digit problems faster than $2 \times 2$-digit problems (5.9 s vs. 9.6 s), $F(1, 59) = 129.19$, and solved problems with 10 as a multiplicand faster than problems without one (5.3 s vs. 10.2 s), $F(1, 59) = 215.9$. There was also a tendency for solutions produced by using the calculator or pencil and paper to be faster than solutions produced by using mental arithmetic (8.0 s vs. 9.8 s), $F(2, 118) = 2.61$, $p = .08$. Mental arithmetic was shown in $t$ tests to be slower than

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<th>Problems and means</th>
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<td>% use</td>
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<td>Structural features</td>
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<td>Strategy means</td>
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<td>Overall means</td>
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*Note.* MA = mental arithmetic; C = calculator; PP = pencil and paper.
either the calculator, $t(59) = 2.89$, or pencil and paper, $t(59) = 3.53$.

All interactions also were significant: Required Strategy × Digits in Multiplicands, $F(2, 118) = 61.74$; Required Strategy × Multiplicand of 10, $F(2, 118) = 104.75$; Multiplicand of 10 × Digits in Multiplicands, $F(1, 59) = 82.19$; and Required Strategy × Multiplicand of 10 × Digits in Multiplicands, $F(2, 118) = 45.27$. As shown in Table 5, the three-way interaction appeared due to the relative speeds of the three strategies varying with the type of problem. On $N \times 10$ and $NN \times 10$ problems, mental arithmetic was fastest, pencil and paper next fastest, and the calculator the slowest. On $N \times NN$ problems, the solution times produced by the three strategies did not vary greatly. On $NN \times NN$ problems, the calculator was fastest, pencil and paper in-between, and mental arithmetic the slowest (exactly the opposite of the pattern on $N \times 10$ and $NN \times 10$ problems).

Accuracy. The older adults solved 1 × 2-digit problems more accurately than 2 × 2-digit ones (6% vs. 12% errors), $F(1, 59) = 41.77$. They solved problems with a multiplicand of 10 more accurately than problems without one (2% vs. 17% errors), $F(1, 59) = 83.68$. Finally, they were most accurate when they used the calculator, next most accurate when they used pencil and paper, and least accurate when they used mental arithmetic, (2%, 8%, and 26% errors), $F(2, 118) = 123.99$.

All interactions also were significant: Required Strategy × Digits in Multiplicands, $F(2, 118) = 51.89$; Required Strategy × Multiplicand of 10, $F(2, 118) = 112.38$; Multiplicand of 10 × Digits in Multiplicand, $F(1, 59) = 55.15$; and Required Strategy × Multiplicand of 10 × Digits in Multiplicands, $F(2, 118) = 54.13$. The three-way interaction reflected accuracy using the calculator being comparatively high on all problems, accuracy with pencil and paper falling off somewhat with increasing problem difficulty, and accuracy with mental arithmetic falling off sharply with problem difficulty (Table 5).

**Overview of Performance in Choice Condition**

**Strategy choices.** As in Experiments 1 and 2, mental arithmetic was used more frequently on problems with 10 as a multiplicand than on ones without them (78% vs. 39%), $F(1, 59) = 219.86$ and on 1 × 2-digit problems than on 2 × 2-digit problems (81% vs. 37%), $F(1, 59) = 139.41$. The interaction of these variables also was significant, $F(1, 59) = 3.9$, $p = .05$, because the difference between 1 × 2- and 2 × 2-digit problems is larger on problems without a 10 than on problems with one (Table 5).

Conversely, participants chose the calculator more often on problems that did not have 10 as a multiplicand than on ones that did (34% vs. 10%), $F(1, 59) = 47.29$, and on 2 × 2-digit problems than on 1 × 2 (34% vs. 9%), $F(1, 59) = 45.83$. These factors interacted, $F(1, 59) = 6.96$, due to the difference in frequency of calculator use on 1 × 2- and 2 × 2-digit problems being greater on problems without a 10.

Finally, the pattern of use for pencil and paper resembled that for the calculator. Participants used pencil and paper more often on problems without 10 as a multiplicand (30% vs. 10%), $F(1, 59) = 37.55$, and more often on 2 × 2-digit problems (27% vs. 13%), $F(1, 59) = 30.25$. The interaction of the two variables was not significant ($F < 1$). In sum, the older adults used mental arithmetic more frequently on easy problems and used pencil and paper or the calculator on more difficult ones.

**Speed.** Relative solution times of the strategies under choice conditions were exactly the opposite of those under no-choice conditions. When participants chose to use mental arithmetic, they solved problems substantially faster than when they chose to use the calculator (3.6 s vs. 10.8 s), $t(43) = 7.99$, or pencil and paper (10.3 s), $t(36) = 10.92$. The time required to use the latter two strategies did not differ. The pattern again reflected participants choosing mental arithmetic disproportionately on the easiest problems.

**Accuracy.** As in the no-choice condition, use of the calculator produced more accurate performance than use of mental arithmetic (3% vs. 8% errors), $t(43) = 2.57$ or use of pencil and paper (13% errors), $t(21) = 10.50$. Unlike in the no-choice condition, however, participants were more accurate in the choice condition when they solved problems using mental arithmetic than when they used pencil and paper, $t(36) = 5.97$. The difference again was attributable to mental arithmetic being used on easier problems in the choice condition than was pencil and paper.

**Tests of ASCM's Predictions**

**Predictors of strategy choices.** A key prediction of ASCM is that differences in performance generated by alternative strategies on a problem should be an excellent predictor of how often each strategy is chosen on that problem, better than any of its structural features. The task of determining the best predictor of strategy choices becomes somewhat more complicated when the choices involve more than two strategies. Strategy use on the four types of problems (Table 5) suggested that participants were making two choices: whether to use mental arithmetic or one of the other two strategies and, when they did not use mental arithmetic, which of the other two strategies to use. Therefore, we conducted two stepwise regression analyses. In the first analysis, the dependent measure was the percentage of use of mental arithmetic on each problem. In the second analysis, the dependent variable was probability of using pencil and paper on each problem, given that either that strategy or the calculator was used. The independent variables included those used in the regression analyses of strategy choices in Experiments 1 and 2, plus four others: difference in mean solution time on the problem between MAR and PPR, difference in mean solution time on the problem between CR and PPR, difference in the percentage of errors on the problem between MAR and PPR, and difference in the percentage of errors on the problem between CR and PPR.

Three factors independently contributed to the percentage of variance accounted for in the percentage of use of mental
arithmetic on each problem: (a) the difference in the percentage of errors on each problem under MAR and CR conditions ($R^2 = .72$); (b) number of digits in the multiplicands (partial $R^2 = .07$); and (c) whether one of the multiplicands was 10 ($R^2 = .06$). Together, the three variables accounted for 85% of the variance in the percentage of use of mental arithmetic on each problem.

Two variables predicted how often the older adults used the calculator, relative to the number of trials on each problem on which they used either it or pencil and paper. The best predictor was again the difference in errors on each problem under MAR and CR conditions ($R^2 = .60$). The greater the discrepancy in errors yielded by mental arithmetic and the calculator, the more likely that participants chose the calculator rather than pencil and paper. Because the percentage of errors under MAR conditions increased sharply with problem difficulty and the percentage of errors under CR conditions did not, this meant that the more difficult the problem, the more likely participants were to choose the calculator rather than pencil and paper. The other significant predictor was whether one of the multiplicands was 10. It accounted for 2% additional, independent variance. Together, the two regression analyses suggested that the older adults used mental arithmetic most often on the easiest problems, the calculator most often on the hardest problems, and pencil and paper most often on problems of intermediate difficulty. This interpretation is congruent with the relative frequency of the strategies (Table 5).

Bias. As in Experiments 1 and 2, participants were slightly biased toward choosing mental arithmetic. On problems where mean solution times of mental arithmetic and the calculator were equal ($± 1$ s), participants used mental arithmetic on 57% of trials.

Effects on solution times of having a choice. The same type of test as in Experiments 1 and 2 was conducted to determine whether being able to choose one’s strategy led to faster solutions, above and beyond the overall frequency with which the strategies were used. As shown in Table 6, the older adults generated answers more quickly when they could choose their strategy, $F(1, 59) = 19.30$. Also as in the previous experiments, they were faster on problems with 10 as a multiplicand, $F(1, 59) = 290.10$. The two variables interacted, $F(1, 59) = 11.68$, because of choice aiding solution times more on problems without 10 as a multiplicand.

Effects on accuracy of having a choice. Choice also led to lower error rates when participants could choose which strategy to use, $F(1, 59) = 17.41$. They also were more accurate on problems with 10 as a multiplicand $F(1, 59) = 84.16$. The significant interaction between the two variables, $F(1, 59) = 28.97$, showed that choice influenced accuracy only on problems that did not have 10 as a multiplicand. On the problems that included 10 as a multiplicand, accuracy was extremely high in both conditions.

Individual differences. On the questionnaire that was presented at the end of the experiment, 20 of the older adults indicated that they had never used a calculator before that day, 14 indicated that they had used calculators but had done so less than once per month both during their working days and during their retirement, and 26 indicated that they had used calculators more often during at least one of the two periods.

Amount of experience proved to be related to how often participants used the calculator. It was used on 19% of trials by the elderly adults who never had used a calculator before the experiment, 29% of trials by those who had used one occasionally, and 30% of trials by those who had used it more than once per month in either their work or current life. Conversely, paper and pencil was used on 29% of trials by those with no experience with calculators, 27% by those with a little experience, and 15% by those with more experience. Putting together these numbers, those older adults who never had previously used a calculator chose it on 40% of trials on which they used either a calculator or pencil and paper; those who had used it less than once per month chose it on 52% of such trials, and those who had used it more than once per month chose it on 67% of such trials.

We also examined the adaptiveness of strategy choices of each of the three experientially defined subgroups. This was done by applying separately to each subgroup the same regression analyses that had been used to examine the choices of all participants in Experiment 3.

The absolute percentages of variance accounted for were lower than for the overall group, not surprisingly because of the reduced sample sizes. However, the basic adaptive quality was the same as for the group as a whole. This held true even for the 20 older adults who had never used the calculator before the experimental session. The best predictor of their percentage of use of the calculator on each problem was the difference between the percentage of errors on the problem in the MAR and CR conditions. It accounted for 51% of the variance in their percentage of use of the calculator on each of the 72 problems. Number of digits in the multiplicands accounted for an additional 14% of the variance, and presence of 10 as a multiplicand added 4% more. The three variables thus accounted for 69% of the variance in the older adults’ frequency of use of the calculator on each problem.

The best predictor of the percentage of choices of the calculator on each problem in both of the other groups of older adults was the difference between solution times in the MAR and CR conditions. It accounted for 36% of the variance for the 14 people in the moderate experience group

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<td>Older Adults' Speed and Accuracy Under Choice and No-Choice Conditions on Problems With and Without a Multiplicand of 10 (Experiment 3)</td>
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<td>Condition</td>
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Note. RT = reaction time.
and for 47% of the variance for the 26 people in the (relatively) high experience group.

Interestingly, amount of experience seemed to have opposite effects on the correlations involving differences in solution times and errors. The correlation between the percentage of calculator use on each problem and the difference between mean solution time on that problem in the CR and MAR conditions steadily increased with calculator experience. It was \( r = .37 \) in the group with no prior experience with calculators, \( r = .60 \) in the group with a little prior experience with them, and \( r = .69 \) in the group with more experience. In contrast, the correlation between the percentage of calculator use on each problem and the difference in the percentage of errors on that problem in CR and MAR groups steadily decreased with calculator experience. It was \( r = .72 \) for the group with no prior experience, \( r = .53 \) for the group with a little prior experience, and \( r = .12 \) for the group with more experience. The findings suggest that when people are first making a strategy choice, they may rely more heavily on its anticipated effects on accuracy, whereas with experience making the choice, they may rely more heavily on anticipated effects on speed.

We next examined the questionnaires to identify differences in attitudes among the three subgroups. The attitudes proved to be quite similar across the groups. On none of the six attitudinal questions did the mean scores differ by more than 0.6 points on the 1 to 7 scale that was used on all questions.

Finally, we considered separately the performance of the 20 older adults who had never used a calculator previously. This examination revealed large individual differences within the group. Of the 20 participants, 7 fit the stereotype of older adults being reluctant to try new strategies, especially ones that involve modern technology. They never used the calculator during the experiment. In contrast, another 7 of the 20 each used the calculator on 100% of trials on which they used either pencil and paper or the calculator. The remaining 6 participants used the calculator on some but not all trials; as a group, they used it on 51% of trials on which they used either pencil and paper or the calculator. The fact that so many older adults who had never used a calculator previously nonetheless used the new technology more often than a technology that they had been using for more than 60 years was not easy to reconcile with the stereotype of older adults in general being reluctant to try new things.

The questionnaire responses of adults in these three subgroups again were similar on most items. In only two cases did mean scores on the 1–7 scale vary by more than a point between those who never used a calculator in the experiment and those who always did (given that they did not use mental arithmetic). The more that participants reported liking calculators and trusting answers calculated mentally, the more often they used the calculator, rather than pencil and paper, in the experiment (means of 4.7, 5.7, and 6.0 for reported liking of calculators and of 4.0, 4.2, and 5.6 for trusting answers computed mentally).

Conclusions

Implications Regarding Adult Cognitive Development

Lemaire and Siegler (1995) distinguished among four types of strategic change: changes in which strategies are used, in their relative frequency of use, in the skill with which they are executed, and in the adaptiveness of choices among them. The present results demonstrated that for the choice between using a calculator or mental arithmetic, some of these factors contribute to changes during adulthood, but others do not.

The one large age-related change was in strategy execution. The no-choice conditions, in which each participant needed to use a given strategy on all problems, yielded particularly clear evidence of this change. Young adults executed the strategies considerably faster and more accurately than did older adults.

In contrast, there were no changes with age in the relative frequency of use of the calculator and mental arithmetic; no changes in the best predictor of the frequency of use of each strategy on each problem; and no changes in how well that predictor accounted for the strategy choices. There was also no change in the type of featural information, presence of 10 as a multiplicand, that accounted for significant additional independent variance; and there were no changes in the existence, direction, and degree of bias in favor of using mental arithmetic rather than the calculator.

In terms of the Lemaire and Siegler (1995) taxonomy, these findings indicate that there were changes in the execution of the strategies but not in which strategies are used, their relative frequency, or the adaptiveness of choices among them. In terms of the distinction within the aging literature between processing deficiency and production deficiency hypotheses (Salthouse, 1991), the results conformed entirely to the processing deficiency interpretation. Because the choice/no-choice method allowed us to obtain independent measures of whether and how often each strategy was used, how well it was executed, and how adaptive the strategy choices were, it provided stronger evidence than previously available that the main locus of strategic change in adulthood, on this task at least, is in the efficiency of strategy execution. A number of previous studies have pointed to the same basic conclusion, but the evidence was obtained with methods that did not allow independent assessment of the different sources of strategic change (e.g., Glyn, Okun, Muth, & Britton, 1983; Rabitt, 1982; Salthouse & Prill, 1987; Salthouse, Legg, Palmon, & Mitchell, 1990; Wright, 1982).

This does not mean that the only source of strategic change during adulthood is efficiency of execution of strategies. A number of previous studies have documented that older and younger adults sometimes use different strategies, choose among them with different degrees of skill, or both (e.g., Adams, Labovitch-Vief, Hobart, & Dorosz, 1990; Charness, 1981, 1987; Cimbalio & Brink, 1982; Reder, Wible, & Martin, 1986; Salthouse, Kaushler, & Saults, 1988). Experiment 3 of the present study illustrated another
situation in which younger and older adults differed in the strategies they used. Older adults used the calculator and pencil and paper equally often. In contrast, none of the 10 younger adults in the pilot study ever chose pencil and paper. When they did not use mental arithmetic, they always used the calculator. The large differences between the two age groups in past history of calculator use seem the likely source of the difference in strategy use. Supporting this interpretation, within the older adult sample, the more pre-experimental experience with calculators that participants had, the more they used the calculator during the experiment.

One of the most striking findings of the study was that contrary to the stereotype of older adults as being set in their ways, a substantial percentage of those who reported never previously having used a calculator nonetheless used it quite often in the experiment. Of the 20 adults who said that they had never before used a calculator, 35% used it on all trials on which they used either a calculator or pencil and paper. An additional 30% used it sometimes (group mean of 52% use of the calculator on trials on which they used either the calculator or pencil and paper). The remaining 35% of the older adults who had never previously used a calculator did conform to the stereotype; they never used the calculator during the experiment.

These findings raise the issue of whether willingness to engage in new activities is a stable dimension of individual differences among older adults. In particular, did the individual differences evident in use of the calculator among those who never had used one previously reflect general differences in inclination to try new activities, moderately general differences in openness to trying new technological devices, or differences specific to the use of calculators? Presenting older adults with a variety of strategy choices, some of which involve new technologies and others of which involve novel nontechnological activities, may indicate the generality of their interest in trying new activities.

**Implications for ASCM**

The present study allowed tests of three central predictions of ASCM. All three predictions were borne out by the data. First, the difference in performance yielded by the strategies on each problem in the no-choice condition was the best predictor of strategy choices on that problem in the choice condition. This prediction held true for both older and younger adults, and it also held true for both two-choice and three-choice situations. The second prediction was that feature information would also influence strategy choices. Consistent with this prediction, presence of 10 as a multiplier added significant independent variance to that which could be accounted for by the performance variables in all of the regression analyses of frequency of use of each strategy on each problem. Again, this held true for both younger and older adults and in both two-choice and three-choice situations. The third prediction was that having a choice would aid performance. In accord with this prediction, speed and accuracy were greater in the choice condition than in the no-choice condition, even when the percentage of use of each strategy in the overall set of problems was statistically equated. Once again, the finding held true for both age groups and both types of choices. In addition to supporting these predictions, the findings also supported a basic assumption of ASCM: that people can accurately estimate the relative times that strategies take to execute and use this information in making strategy choices.

The results also pointed to a way in which ASCM needs to be augmented. In all three experiments, participants chose mental arithmetic more often than justified by the performance it yielded. The bias was not huge—about 1.5 s—but it was consistent in both direction and magnitude for both age groups and both types of choices.

What might lead to such a bias? One plausible source is self-evaluation: People might think better of themselves when they solve problems in their head, rather than when they have to resort to an external aid. Another plausible source is external evaluation: Participants may have wanted to impress the experimenter. A third possibility is that people may find it more aesthetically pleasing to solve problems mentally. Finally, the subjective ease of processing may not be perfectly indexed by the amount of time the processing takes. When problem solving takes the same amount of time, mental arithmetic may feel easier than using an external device. Clearly these possibilities are not mutually exclusive; any or all could contribute to the bias.

The more general lesson is that factors other than speed and accuracy influence strategy choices. ASCM already embodies one such factor: novelty. The system is biased to use new strategies more often than their speed and accuracy would justify. Because execution of new strategies usually is far worse than will later be the case, this allows the system to gain sufficient experience with the new approaches to estimate their performance characteristics. The present results indicate that ASCM needs to be opened up further to encompass the characteristics that account for biases, such as the preference for mental arithmetic observed in this study.

**Methodological Implications**

This study illustrated several ways in which the choice/no-choice method is useful for studying strategic development. The core advantage, from which the others follow, is that the method yields unbiased estimates of the performance characteristics of strategies. Consider evidence for this advantage. In the choice condition of all three experiments, solution times were faster when participants used mental arithmetic than when they used the calculator. In typical experiments, in which participants are free to choose their strategy on all trials, this would likely have led to the conclusion that the strategy that yielded the faster times is the faster strategy. Performance in the no-choice conditions, however, showed that this conclusion would have been wrong. When each strategy was used equally often on all problems and by all participants, the results were reversed. The calculator was faster than mental arithmetic, as was pencil and paper when that was included as an option.
Such selection artifacts are not limited to the present experimental context; rather, they seem likely to be extremely pervasive. In all studies of strategy choice that we have encountered, participants use strategies most often on the problems on which those strategies are most effective, relative to available alternatives. Strategies that vary greatly in the accuracy they generate on different problems, such as retrieval and mental calculation, are generally chosen especially often on the easiest problems. Similarly, participants who are especially competent in a domain generally use such strategies the most often, because they can execute them accurately on a broader range of problems than can other participants. Both of these phenomena lead to the speeds and accuracies of such strategies under free-choice conditions being faster than they would be if the strategies were used on a representational set of problems and by a representative set of participants. Only by eliminating such selection biases can accurate estimates of the performance properties of strategies be obtained.

Obtaining unbiased estimates of these performance properties of strategies is critical for several purposes. One is to test theories of choice. The unbiased estimates obtained in the no-choice conditions of the present experiments were crucial for testing ASCM’s theoretical prediction that the best predictor of frequency of choices of a given strategy would be the performance that the strategy yields relative to the performance yielded by alternative approaches. The unbiased estimates also were crucial for testing the prediction that structural features of problems, such as presence of 10 as a multiplicand, would influence choices above and beyond the influence of the relative effectiveness of the strategies. Without estimates of the strategies’ relative effectiveness, there would then be no way to test this latter prediction. The unbiased estimates of speeds and accuracies yielded by the strategies also were indispensable for determining the degree of bias in the choices. Again, without a precise estimate of the speed and accuracy that each strategy yielded on each problem, such estimates of bias would seem impossible.

The importance of unbiased estimates of performance characteristics is equally great for evaluating predictions of other models. One prominent example is Lovett and Anderson’s (in press) ACT-R model of strategy choice. ACT-R predicts that strategy choices should be an additive function of problem characteristics and of each person’s history of success and failure with each strategy. Without an unbiased estimate of the performance yielded by each strategy on comparable problems, it is impossible to determine whether people’s choices reflect their direct experience with the strategies or their projecting the performance each strategy would yield on an unbiased set of problems. ACT-R predicts the former; ASCM the latter. By providing data on the person’s actual experience (from the choice condition) and unbiased estimates of problem difficulty (from other people’s performance on identical problems in the no-choice conditions), the choice/no-choice method could yield the type of data needed to distinguish between the two theoretical predictions.

To what range of tasks can the choice/no-choice method be applied? Two characteristics seem essential. One is that strategy use can be assessed reliably on a problem-by-problem basis. The other is that it be possible to determine if participants in the no-choice conditions are using the strategy that they are supposed to be using.

A variety of tasks and strategy choices meet these requirements. For example, two domains on which it has already been shown that strategies can be assessed reliably are serial recall (McGilly & Siegel, 1990) and spelling (Siegel, 1986). The choice/no-choice method could be applied to each of these. Serial recall might be studied by examining performance when participants are free to rehearse or not rehearse, when they need to rehearse aloud on each trial during the waiting period, and when they need to engage in an activity that precludes rehearsal on each trial during the waiting period. Studies of spelling could compare performance when participants are free to choose whether to look up a word in the dictionary, when they need to spell the word from memory on each trial, and when they need to look up the word in a dictionary on each trial.

The method could also be used to examine choices among alternative study strategies. At first glance, it would appear that the best strategy for deciding what to study would be to focus attention on the least known material. However, this may not be the case: in at least one experiment, 9-year-olds who focused on items they previously had not remembered did no better than those who studied a broader range of items (Masur, McIntyre, & Flavell, 1973). This might mean that focusing on difficult items is ineffective, but another possibility is that the children who chose this strategy were more likely than others to have been to forget the items that they did not study. The choice/no-choice method provided a way of addressing this issue. In particular, learning and memory could be examined when participants are free to decide how much time to devote to studying each item, when they need to spend equal amounts of time on each item, and when they need to focus entirely on the items that elicited errors on the previous trial.

A quite different domain in which strategy choices could be examined is haptic exploration of objects. Haptic processing is usually studied by asking people to explore objects manually in order to be able to later recognize them visually (e.g., Lederman & Klatsky, 1987). It is unknown, however, whether the exploration strategies contribute to their success at this task. Exploration of objects of varying complexity in a choice condition could be compared to alternative no-choice exploration methods, such as requiring exploration to be conducted with a particular hand (and subsequently recognized with a particular eye) or requiring exploration of the surfaces to be conducted in a particular order. Such experiments could not only assess the benefits of choice in this area but also address such issues as whether there is hemispheric specialization for haptic processing (Fagot, Lacourse, & Vauclair, 1993; Summers & Lederman, 1990).

Like all methods, the choice/no-choice approach is limited in its range of applicability. It would not be effective for studying populations whose strategy use cannot be influenced by verbal instructions. For example, infants choose
adaptively among alternative strategies for descending down ramps (Adolph, 1995), but telling them in a no-choice condition to always go down using a particular method would not be effective. The method also requires means for producing valid strategy assessments on a trial-by-trial basis. This requirement will more often be met on tasks where the strategies are discrete and easily separated, such as arithmetic, than on tasks where the strategies blend into each other, such as reading comprehension. Another limit is that the method requires tasks on which sufficient numbers of trials per item can be run to establish temporal and accuracy characteristics of the strategies. This precludes its being used with tasks on which each trial takes a long time. Finally, it is easier to use the method on tasks on which overt behavior can be used to validate strategy use than on ones on which it cannot. The overt behavior need not accompany strategy use ordinarily, as long as it can be produced in response to the no-choice instructions. Thus, adults may not ordinarily rehearse overtly on serial-recall tasks, but as long as they can do so when requested to rehearse aloud, the method will be effective.

These limits mean that the choice/no-choice method cannot be used to study all strategy choices with all populations. On the numerous tasks and populations on which it can be used, however, it offers a means for obtaining unbiased assessments of the performance characteristics of strategies. These unbiased estimates provide the data needed to compare predictions of alternative theories of strategy choice, to determine the degree to which strategy choices are influenced by the strategies’ performance characteristics, and to demonstrate the influence of other variables that contribute to strategy choices. In sum, the choice/no-choice method yields the type of data needed to advance understanding of the strategy choice process.

References


OLDER AND YOUNGER ADULTS' MULTIPLICATION


(Appendixes follow on next page)
Appendix A
Problems Used in Experiment 1

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<th>Subset 1</th>
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| 2 × 2-digit problems |
| 10 × 12   | 10 × 17   | 10 × 14   |
| 10 × 16   | 23 × 10   | 10 × 18   |
| 56 × 10   | 10 × 49   | 54 × 10   |
| 60 × 10   | 55 × 10   | 58 × 10   |
| 11 × 12   | 11 × 11   | 11 × 14   |
| 11 × 18   | 15 × 20   | 20 × 11   |
| 12 × 15   | 15 × 11   | 12 × 15   |
| 13 × 20   | 12 × 20   | 12 × 20   |
| 14 × 18   | 12 × 19   | 14 × 17   |
| 13 × 18   | 13 × 18   | 13 × 19   |
| 25 × 15   | 24 × 16   | 24 × 16   |
| 23 × 17   | 22 × 18   | 26 × 14   |

Note. Five problems (5 × 10, 12 × 15, 12 × 20, 13 × 18, and 24 × 16) were inadvertently presented in two different subsets. However, when analyses without these problems were carried out, they yielded very similar results. In Experiment 2, these problems were replaced in one set by the same problems with the multipliers reversed (5 × 10 replaced by 10 × 5).

Appendix B
Problems Used in Experiment 3

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| 2 × 2-digit problems |
| 54 × 10   | 58 × 10   | 56 × 10   | 60 × 10   |
| 10 × 22   | 23 × 10   | 10 × 49   | 49 × 10   |
| 10 × 18   | 10 × 16   | 10 × 14   | 10 × 12   |
| 26 × 14   | 22 × 18   | 24 × 16   | 16 × 24   |
| 12 × 20   | 13 × 20   | 15 × 20   | 20 × 11   |
| 23 × 17   | 25 × 15   | 14 × 17   | 13 × 21   |
| 14 × 18   | 13 × 18   | 11 × 18   | 18 × 13   |
| 12 × 15   | 15 × 11   | 15 × 12   | 11 × 12   |
| 12 × 19   | 11 × 13   | 11 × 11   | 19 × 12   |

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